Revealed preference and satisficing behavior

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Abstract. A much discussed topic in the theory of choice is how a preference order among options can be derived from the assumption that the notion of 'choice' is primitive. Assuming a choice function that selects elements from each finite set of options, Arrow (1959) already showed how we can generate a weak ordering by putting constraints on the behavior of such a function such that it behaves as a utility maximizer. Arrow proposed that rational agents can be modeled by such choice functions. Arrow's standard model of rationality has been criticized in economics and gave rise to approaches of *bounded rationality.* Two standard assumptions of rationality will be given up in this paper. First, the idea that agents are utility *optimizers* (Simon). Second, the idea that the relation of 'indifference' gives rise to an equivalence relation. To account for the latter, Luce (1956) introduced semi-orders. Extending some ideas of Van Benthem (1982), we will show how to derive semi-orders (and so-called interval orders) based on the idea that agents are utility *satisficers* rather than utility optimizers.

Keywords: Revealed preference, Satisficing behavior, comparatives, vagueness

1. Deriving weak orderings

A much discussed topic in the theory of choice is how a preference order among options can be derived on the assumption that the notion of *choice* is primitive. Assuming a choice function that selects elements from each finite set of options, Arrow (1959) already showed how we can generate a weak ordering by putting constraints on how this function should behave on different sets of options. Let us define a *choice structure* to be a triple $\langle A, O, C\rangle$, where A is a non-empty set of actions, the set O consists of all finite subsets of A, and the choice function C assigns to each finite set of options $o \in O$ a subset of o, $C(o)$. Arrow (1959) stated the following principle of choice (C), and the constraints (A1) and (A2) to assure that the choice function behaves in a 'consistent' way:

(C) $\forall o \in O : C(o) \neq \emptyset$.
(A1) If $o \subseteq o'$, then $o \cap C(o') \subseteq C(o)$.
(A2) If $o \subseteq o'$ and $o \cap C(o') \neq \emptyset$, then $C(o) \subseteq C(o')$.

If we say that $x > y$, iff$_{def}$ $x \in C(\{x, y\}) \wedge y \notin C(\{x, y\})$, one can easily show that the ordering as defined above gives rise to a *weak order*. A structure $\langle I, R\rangle$, with R a binary relation on I, is a weak order just in case R is irrreflexive (IR), transitive (TR), and almost connected (AC).[1]

DEFINITION 1. *A weak order is a structure $\langle I, R\rangle$, with R a binary relation on I that satisfies the following conditions:*
(IR) $\forall x : \neg R(x, x)$.
(TR) $\forall x, y, z : (R(x, y) \wedge R(y, z)) \rightarrow R(x, z)$.
(AC) $\forall x, y, z : R(x, y) \rightarrow (R(x, z) \vee R(z, y))$.

If we now define the indifference relation, '$\sim$', as follows: $x \sim y$ iff$_{def}$ neither $x > y$ nor $y > x$, it is clear that '$\sim$' is predicted to be an equivalence relation. It is well-known that in case '$>$' gives rise to a weak order, it can be represented numerically by a real valued function u such that for all $x, y \in I$: $x > y$ iff $u(x) > u(y)$, and $x \sim y$ iff $u(x) = u(y)$.

Within the standard model of economics, an agent is taken to be a 'rational man' if his behavior can be abstractly described in terms of a choice function that satisfies conditions (C), (A1), and (A2).

This standard model has been criticized in economics and gave rise to approaches of *bounded rationality*. Two standard assumptions of rationality will be given up in this paper. First, the idea that agents are utility *optimizers*. Second, the idea that the relation of 'indifference' gives rise to an equivalence relation. As for the first, the notion of 'rational man' came under attack in the writings of Herbert Simon in the 1950s. He claimed that individuals do not necessarily look for the *best* alternative(s) in a feasible set, but rather that they accept alternatives which they consider satisfactory, i.e., they have a *satisficing* rather than an *optimizing* behavior. Tyson (2003) argues that to account for Simon's satisficing behavior, we have to give up on axiom (A1). His argument is that (A1) implements the cognitive assumption that the decision maker fully perceives his preferences among available alternatives. Another way of thinking of Simon's criticism is that we should not seek to derive the meaning of 'better than' in terms of the meaning of 'best' – as is assumed if agents are taken to be utility maximizers –, but rather to derive the meaning of 'better than' in terms of the context-dependent meaning of 'good'.[2] What is crucial for the interpretation of the results of our paper is that although 'good' seems to obey axiom (A2), axiom (A1) seems much too strong: (A1)

[1] In the economic literature, the property of being almost connected is normally called *negative transitivity* and stated as follows: $\forall x, y, z : (\neg R(x, z) \wedge \neg R(z, y)) \rightarrow \neg R(x, y)$. Obviously, this is just equivalent with (AC).

[2] Interestingly enough, this is exactly analogue to what Klein (1980) intended to do in linguistics: the meaning of 'taller than' (or 'better than') should be defined in terms of the meaning of 'tall' (or 'good'), not that of 'tallest' (or 'best').

demands that if both x and y are considered to be good in the context of $\{x, y, z\}$, both should considered to be good in the context $\{x, y\}$ as well. But that is exactly what we don't want for a context dependent notion of 'good':[3] in the latter context, we want it to be possible that only x, or only y, is considered to be good. We should conclude that if we want to characterize the behavior of 'good', we should give up on (A1). Unfortunately, by just constraints (C) and (A2) we cannot guarantee that the comparative relation 'better than' behaves as desired. In particular, we cannot guarantee that it behaves almost connected.

To assure that the comparative behaves as desired, we add to (C) and (A2) the Upward Difference-constraint (UD), proposed by Van Benthem (1982). To state this constraint, we define the notion of a difference pair: $\langle x, y\rangle \in D(o)$ iff$_{def}$ $x \in C(o)$ and $y \in (o - C(o))$. Now we can define the constraint:

(UD) $o \subseteq o'$ and $D(o') = \emptyset$, then $D(o) = \emptyset$.

In fact, van Benthem (1982) states the following constraints: No Reversal (NR), Upward Difference (UD), and Downward Difference (DD) (where o^2 abbreviates $o \times o$, and $D^{-1}(o) =_{def} \{\langle y, x\rangle : \langle x, y\rangle \in D(o)\}$):

(NR) $\forall o, o' \in O : D(o) \cap D^{-1}(o') = \emptyset$.
(UD) $o \subseteq o'$ and $D(o') = \emptyset$, then $D(o) = \emptyset$.
(DD) $o \subseteq o'$ and $D(o) = \emptyset$, then $D(o') \cap o^2 = \emptyset$.

Van Benthem (1982) shows that if constraints (NR), (UD) and (UD) are satisfied, the relations '$\sim$' and '$>$' as defined before still have the same properties as before: '$\sim$' is still predicted to be an equivalence relation, while the relation '$>$' is still predicted to be (i) irreflexive, (ii) transitive, and (iii) almost connected. It is almost immediate that in case C picks *an element* of each $o \in O$, instead of a subset, the resulting ordering will also be *connected* (satisfy for each $x, y \in I : x > y, y > x$, or $x = y$) and thus be a *linear order*.

2. Semi orders and Interval orders

The 'indifference'-relation induced by the preference order derived in the previous section is predicted to be transitive, just like the indifference relation induced by preference orders on which the standard theory

[3] You might interpret 'satisficing behavior' in another way: x is good, if it meets some fixed, context independent criteria of acceptability. I find this context independent notion of 'good' rather boring, however.

of choice is based. Already in the 1930s, economists like Armstrong claimed that a basic assumption of the classic utility model, namely, the transitivity of the indifference relation, is highly arguable. A well-known version of this examle is due to Luce (1956):

> A person may be indifferent between 100 and 101 grains of sugar in his coffee, indifferent between 101 and 102, ..., and indifferent between 4999 and 5000. If indifference were transitive he would be indifferent between 100 and 5000 grains, and this is probably false.

It is clear that the non-transitivity of indifference results from the inability of human beings to discriminate close quantities. In fact, problems of such kind were already discussed by philosophers in ancient Greece in the so-called 'paradox of the heap', the famous problem induced by vague expressions.

Luce (1956) accounted for the intransitivity of indifference or indiscrimination relations according to which the individual has an (ordinal) real-valued function u defined on the set of alternatives and there exists a given non-negative quantity ϵ called the *treshold*. When the individual has to choose from a subset c of feasible alternatives, he chooses the alternative y such there does not exist an x with $u(x) > u(y) = \epsilon$. This model is called a *treshold utility* model. As above, there exists an equivalent model based on a preference relation. This preference relation is not a weak order, but rather what Luce (1956) calls a *semi-order*. A structure $\langle I, R\rangle$, with R a binary relation on I, is a semi-order just in case R is irrreflexive (IR), semitransitive (STr), and satisfies the interval-order (IO) condition. A structure that satisfies (IR) and (IO) is called an *Interval order*. A (strict) *partial order* is, of course, an order that is irreflexive and transitive.

DEFINITION 2. *A semi order is a structure $\langle I, R\rangle$, with R a binary relation on I that satisfies the following conditions:*
(IR) $\forall x : \neg R(x, x)$.
(IO) $\forall x, y, v, w : (R(x, y) \wedge R(v, w)) \rightarrow (R(x, w) \vee R(v, y))$.
(STr) $\forall x, y, z, v : (R(x, y) \wedge R(y, z)) \rightarrow (R(x, v) \vee R(v, z))$.

DEFINITION 3. *An Interval order is a structure $\langle I, R\rangle$, with R a binary relation on I that satisfies the following conditions:*
(IR) $\forall x : \neg R(x, x)$.
(IO) $\forall x, y, v, w : (R(x, y) \wedge R(v, w)) \rightarrow (R(x, w) \vee R(v, y))$.

It is easy to see that if $\langle I, R\rangle$ is an interval order, $\langle I, R\rangle$ is a (strict) partial order as well. But this means that weak order are stronger than semi-order, which are stronger than interval orders, which in turn are stronger than strict partial orders. Semi-orders are obviously relevant for economics, psychology, but also for linguistics and philosophy as it deals with the analysis of vagueness. The same is true for Interval orders, which are argued also to be relevant for the representation of events (e.g. Wiener, 1914; Thomason, 1984). Just like Arrow (1959) and van Benthem (1982) did for weak orders, it would be nice if we could characterize semi-orders and interval orders in terms of the behavior of optimal (Arrow) or satisficing (van Benthem) choice functions among sets of options. Fishburn (1975) already showed how to solve the former problem. It is still an open issue, however, how to derive semi-orders (and interval orders) in terms of satisficing rather than optimizing behavior. The main aim of this paper is to solve this open issue.

3. Deriving the orders

Our derivation/characterization of several preference orders makes use of *two choice functions*, that intuitively pick the *good* and the *bad* elements. Let us say that $C(o)$ selects the elements of o that are (clearly) good, while $\overline{C}(o)$ selects the elements that are (clearly) bad. Let us define the pairs of elements of o of which the first elements is good and the second element bad by $D_{C\overline{C}}(o) =_{def} \{\langle x, y\rangle : x \in C(o) \wedge y \in \overline{C}(o)\}$, and similarly for $D_{\overline{C}C}(o)$. We also define $D_{CN}(o)$ to be the set of ordered pairs of which the first elements is good and the second element is neither good nor bad: $D_{CN}(o) =_{def} \{\langle x, y\rangle : x \in C(o) \wedge y \notin C(o) \wedge y \notin \overline{C}(o)\}$, and similarly for $D_{NC}(o)$, $D_{N\overline{C}}(o)$, and $D_{\overline{C}N}(o)$. In terms of these notions, we can define the set of upward and downward difference pairs:

$D^{\uparrow}(o) =_{def} D_{C\overline{C}}(o) \cup D_{CN}(o) \cup D_{N\overline{C}}(o)$.
$D^{\downarrow}(o) =_{def} D_{\overline{C}C}(o) \cup D_{NC}(o) \cup D_{\overline{C}N}(o)$.

Now we can give the following four constraints:

(C*) $\forall o \in O : C(o) \cap \overline{C}(o) = \emptyset$.
(NR*) $\forall o, o' : D^{\uparrow}(o) \cap D^{\downarrow}(o') = \emptyset$.
(UD*) If $o \subseteq o'$ and $D_{C\overline{C}}(o') = \emptyset$, then $D_{C\overline{C}}(o) = \emptyset$.
(DD*) If $o \subseteq o'$ and $D_{C\overline{C}}(o) = \emptyset$, then $D_{C\overline{C}}(o') \cap o^2 = \emptyset$.

Constraints (UD*) and (DD*) are very similar to the earlier Upward and Downward Difference constraints of van Benthem (1982), while (C*) assures that C and $\overline{C}$ behave as contraries. The crucial difference with van Benthem's characterization of weak orders is due to the No Reversal constraint (NR*), which is much weaker now, due to our use of two, instead of one choice function. We define the preference relation as follows: $x > y$ iff$_{def}$ $x \in C(\{x, y\})$ and $y \in \overline{C}(\{x, y\})$. Then we can prove that the preference relation behaves irreflexive and transitive,[4] but it need *not* satisfy *almost connectedness*: If $x > y$, it is possible that neither $x > z$ nor $z > y$, because (DD*) doesn't require either of them to hold if $C(\{x, y, z\}) = \{x\}$ and $\overline{C}(\{x, y, z\}) = \{y\}$.

Now we can prove the following theorem:

THEOREM 1. *Any choice structure $\langle A, O, C, \overline{C}\rangle$ with A and O as defined above such that C and $\overline{C}$ obey axioms (C*), (NR*), (UD*), and (DD*), gives rise to a semi order $\langle A, >\rangle$, if we define $x > y$ as $x \in C(\{x, y\})$ and $y \in \overline{C}(\{x, y\})$.*

In order to show that the above constraints guarantee that the induced ordering is a semi-order, we have to show that (IO) and (STr) hold.

Proof of (IO): Assume $x > y$ and $v > w$. Because $x > y$, the following constellations are possible for $\langle x, y, w\rangle$ (with (NR*) and (UD*)): $C\overline{CC}, CC\overline{C}, CN\overline{C}$, $C\overline{C}C, N\overline{C}C, \overline{CC}C$, and $C\overline{C}N$. From the first three options, we immediately conclude with (DD*) that $x > w$. From the fourth, fifth, and sixth options, we conclude that $w > y$, from which we can easily derive with $v > w$ and transitivity that $v > y$. Thus, the only possible constellation that doesn't satisfy the consequent is $C\overline{C}N$. By parallel reasoning, the only possible constellation for $\langle v, w, y\rangle$ that doesn't satisfy the consequent of (IO) is $C\overline{C}N$ (because $v > w$). Thus, the only constellation that doesn't satisfy (IO) at all is where both $\langle x, y, w\rangle$ and $\langle v, w, y\rangle$ behave as $C\overline{C}N$. But now $\langle y, w\rangle \in D^{\uparrow}(\{v, w, y\})$ and $\langle y, w\rangle \in D^{\downarrow}(\{x, y, w\})$, which is ruled out by (NR*).

The above reasoning shows that the induced ordering gives rise to an *interval-order*. On top of that, the ordering will also be a *semi-order* if it also satisfies semi-transitivity, (STr). We prove this as follows: Asume $x > y$ and $y > z$. Because $x > y$, the following constellations are possible for $\langle x, y, v\rangle$ (with (NR*) and (UD*)): $C\overline{CC}, CC\overline{C}, CN\overline{C}, C\overline{C}C$, $N\overline{C}C, \overline{CC}C$, and $C\overline{C}N$. From the first three options we immediately conclude with (DD*) that $x > v$. From the fourth, fifth, and sixth options, we conclude with $y > z$ and transitivity that $v > y$, from which we can easily derive that $v > z$. Thus, the only possible constellation that doesn't satisfy the consequent of (STr) is $C\overline{C}N$. By

[4] Irreflexivity follows immediately from the definition of the comparative together with constraint (C*). Transitivity can be proved as follows: Suppose $x > y$ and $y > z$, now look at $\langle x, y, z\rangle$. Then we have the following possibilities not ruled out by (UD*): $C\overline{C}C, C\overline{C}N, \overline{CC}C, \overline{C}NC, N\overline{C}C$, and $NC\overline{C}$, which are all in contradiction with (NR*), and $CC\overline{C}, C\overline{CC}$, and $CN\overline{C}$, from which we can derive via (DD*) that $x > z$. It follows that $x > z$, which means that $>$ is transitive.

parallel reasoning, the only constellation for $\langle y, z, v\rangle$ is $C\overline{C}N$ (because $y > z$). But now $\langle y, v\rangle \in D^{\downarrow}(\{x, y, v\})$ and $\langle y, v\rangle \in D^{\uparrow}(\{y, z, v\})$, which is ruled out by (NR*). Conclusion: the comparative generated by our four constraints gives rise to a semi-order.

It is also possible to show something stronger than theorem 1, i.e., that our constraints in fact *characterize* semi-orders. To do that, we have to show that from each semi-order we can define a choice structure $\langle A, O, C, \overline{C}\rangle$ that satisfies the above constraints. Let $\langle I, R\rangle$ be a semi-order. We define $\langle A, O, C, \overline{C}\rangle$ as follows: A is the old set and O is the totality of its finite subsets. For the definitions of C and $\overline{C}$, we look for each $o \in O$ at the top- and bottommost pairs of objects in o that stand in the '$>$'-relation. Choice function C picks the topmost object(s) in o, while $\overline{C}$ picks the bottommost objects in o. In case each pair of objects in o are all mutually $\sim$-related, $C(o) = o$ (or $\overline{C}(o) = o$.) It is easy to see that the generated structure satisfied the constraints.

In terms of two instead of one choice functions, we can also derive/characterize other ordering relations. In order to do so, we define the following new constraints (C**), (NR**), and (UD**):

(C**) $\forall o \in O : C(o) \cap \overline{C}(o) = \emptyset$ and $C(o) \cup \overline{C}(o) = o$.
(NR**) $\forall o, o' \in C : a, b \in \{C, \overline{C}, N\} : a \neq b \rightarrow D_{ab}(o) \cap D_{ba}(o') = \emptyset$.
(UD**) If $o \subseteq o'$ and $\langle x, y\rangle \in D_{C\overline{C}}(o)$, then $D_{C\overline{C}}(o') \neq \emptyset$ and $\langle x, y\rangle \notin D^{\downarrow}(o)$.

Notice that (C**) and (UD**) strengthen (C*) and (UD)/(UD*) respectively, while (NR**) is stronger than (NR), but weaker than (NR*). Noitice also that in the context of (NR*), (UD**) reduces to (UD)/(UD*). Now we can state the following theorems:

THEOREM 2. *Any choice structure* $\langle A, O, C, \overline{C}\rangle$ *with* A *and* O *as defined above such that* C *and* $\overline{C}$ *obey axioms (C*), (NR), (DD) and (UD**), gives rise to a (strict) partial order* $\langle A, >\rangle$*, if we define* $x > y$ *as* $x \in C(\{x, y\})$ *and* $y \in \overline{C}(\{x, y\})$.

THEOREM 3. *Any choice structure* $\langle A, O, C, \overline{C}\rangle$ *with* A *and* O *as defined above such that* C *and* $\overline{C}$ *obey axioms (C*), (UD*), (DD*) and (NR**), gives rise to an interval order* $\langle A, >\rangle$*, if we define* $x > y$ *as* $x \in C(\{x, y\})$ *and* $y \in \overline{C}(\{x, y\})$.

THEOREM 4. *Any choice structure* $\langle A, O, C, \overline{C}\rangle$ *with* A *and* O *as defined above such that* C *and* $\overline{C}$ *obey axioms (C**), (NR*), (UD*), and (DD*), and where gives rise to a weak order* $\langle A, >\rangle$*, if we define* $x > y$ *as* $x \in C(\{x, y\})$ *and* $y \in \overline{C}(\{x, y\})$.

The proofs of these theorems are relatively simple. To prove theorem 4, for instance, it is enough to observe that in case (C**) holds, the constraints (NR*), (UD*), and (DD*) reduce to (NR), (UD), and (DD), and we are back to van Benthem's characterization of weak orders.

In order to prove theorem 2, we have to show that $>$ is irreflexive and transitive. The former is true by definition, and to prove the latter, assume $x > y$ and $y > z$. Given the constraints (C*), (NR), (DD) and (UD**), this means that the following constellations are possible for $\langle x, y, z\rangle$: $C\overline{CC}, CC\overline{C}, CN\overline{C}$.[5] From all three and (DD*) we conclude that $x > z$. Notice that the constraints do not guarantee that the interval order condition (IO), semi transitivity (STr), and almost connectedness (AC) are obeyed. (IO): if $x > y$ and $v > w$ it is possible for constellation $\langle xyw\rangle$ to be $C\overline{C}N$ and for constellation $\langle vwy\rangle$ to be $C\overline{C}N$ without deriving a contradiction. (STr): if $x > y$ and $y > z$, it is possible for constellations $\langle xyv\rangle$ and $\langle yzv\rangle$ to be $C\overline{C}N$ without deriving a contradiction. (AC): if $x > y$ it is possible for constellation $\langle xyv\rangle$ to be $C\overline{C}N$ without deriving a contradiction.

To prove theorem 3, we have to prove that (IO) holds. So suppose $x > y$ and $v > w$. Because $x > y$, the following constellations are possible for $\langle x, y, w\rangle$ (with (NR**) and (UD*)): $C\overline{CC}, CC\overline{C}, CN\overline{C}$, $C\overline{C}C, N\overline{C}C, \overline{CC}C$, and $C\overline{C}N$. From the first three options, we immediately conclude with (DD*) that $x > w$. From the fourth, fifth, and sixth options, we conclude that $w > y$, from which we can easily derive with $v > w$ and transitivity that $v > y$. Thus, the only possible constellation that doesn't satisfy the consequent is $C\overline{C}N$. By parallel reasoning, the only possible constellation for $\langle v, w, y\rangle$ that doesn't satisfy the consequent of (IO) is $C\overline{C}N$ (because $v > w$). Thus, the only constellation that doesn't satisfy (IO) at all is where both $\langle x, y, w\rangle$ and $\langle v, w, y\rangle$ behave as $C\overline{C}N$. But now $\langle y, w\rangle \in D_{\overline{C}N}(\{v, w, y\})$ and $\langle y, w\rangle \in D_{N\overline{C}}(\{x, y, w\})$, which is ruled out by (NR**). Notice that the constraints do not guarantee that semi transitivity (STr) and almost connectedness (AC) are obeyed. (STr): if $x > y$ and $y > z$, it is possible for constellations $\langle xyv\rangle$ and $\langle yzv\rangle$ to be $C\overline{C}N$ without deriving a contradiction. (AC): if $x > y$ it is possible for constellation $\langle xyv\rangle$ to be $C\overline{C}N$ without deriving a contradiction.

4. Conclusion and outlook

In this paper I have given a derivation of the meaning of 'better than' from a plausible analysis of the meaning of 'good', instead of the mean-

[5] Notice that constellations $C\overline{C}N$ and $NC\overline{C}$ are ruled out by (UD**).

ing of 'best'. I have argued that the *interpretation* of these technical results is relevant for the modeling of *bounded rational* agents. Semi-orders and interval orders are relevant in this respect because they were introduced by Luce (1956) to model the fact that behavior is invariant on just noticeable differences in preference. The axioms that constrain the behavior of the choice functions that models the meaning of 'good' (and 'bad') is argued to be an interpretation of Simon's *satisficing*, rather than utility maximizing agents. On the basis of this interpretation, this paper gives a characterizations of several preference relations in terms of constraints on the behavior of satisficing agents.

In this paper I interpreted the relations '$>$' and '$\sim$' as preference and indifference, respectively. But there is nothing in the formal analysis that requires us to interpret the relations in that way. Two other equally natural interpretations are 'taller than' and 'equally tall as', and 'earlier/later than' and 'simultaneous with'. In fact, the original motivation of this work came from the latter two interpretations (see Van Rooij, to appear). I believe that the results of this paper are significant as well for the analysis of natural language comparatives, temporal relations, and more generally, for the analysis of vagueness.

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