Some Polyadic Quantifiers of Natural Language

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April 4, 2008

1 Introduction

Definition 1 *A generalized quantifier* Q *of type* $(n_1, \ldots, n_k)$ *is a functor assigning to every set* M *a* k*-ary relation* $\mathsf{Q}(M)$ *between relations on* M *such that if* $(R_1, \ldots, R_k) \in \mathsf{Q}(M)$ *then* R_i *is an* n_i*-ary relation on* M*, for* $i = 1, \ldots, k$*. Additionally* Q *is preserved by bijection.*

If for all i: $n_i \leq 1$, then we say that quantifier is *monadic*, otherwise we call it *polyadic*.

2 What Do Reciprocals Mean?

Consider the following sentences:

(1) Parliament members refer to each other indirectly.

(2) Boston pitchers sat alongside each other.

(3) Pirates were staring at each other in surprise.

(4) Stones are arranged on top of each other.

(5) Planks were stacked atop of each other.

Typical models satisfying these sentences are illustrated in Figures 1 and 2.

2.1 Reciprocals as Polyadic Quantifiers

Dalrymple et al. (1998) noticed that we can treat reciprocal expressions as polyadic quantifiers of type (1, 2) ($\mathsf{EACH\ OTHER}(A, R)$) which meanings can be characterized by 2 parameters.

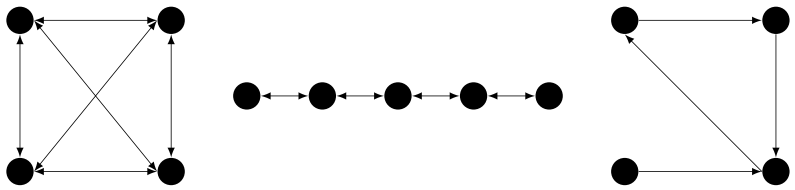

Figure 1: On the left, model satisfying sentence (1). This is so-called *strong reciprocity*. Each element is related to each of the other elements. In the middle, model satisfying sentence (2). This is *intermediate reciprocity*. Each element is related to each other element by a chain of relations. On the right, model satisfying sentence (3), so-called *weak reciprocity*. For each element there exists a different related element.

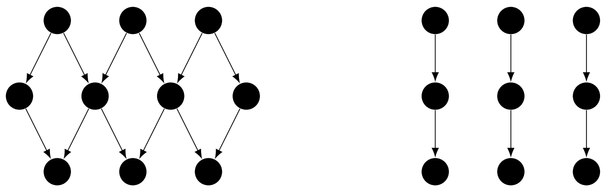

Figure 2: On the left model, satisfying sentence (4), so-called *intermediate alternative reciprocity*. Ignoring the direction of arrows, every element is connected directly or indirectly. On the right model satisfying sentence (5), so-called *weak alternative reciprocity*. Each element participates with some other element in the relation as the first or as the second argument, but not necessarily in both roles.

The first one relates to how the scope relation R should cover the domain.

FUL Each pair of elements from A participates in R directly.

LIN Each pair of elements from A participates in R directly or indirectly.

TOT Each element in A participates in R with at least one other element.

The second parameter determines whether the relation R between individuals in A is the extension of the reciprocal's scope (R), or is obtained from the extension by ignoring the direction in which the scope relation holds ($R^{\vee} = R \cup R^{-1}$).

By combining these 2 parameters Dalrymple et al. (1998) got 6 possible meanings for reciprocals: strong reciprocity (FUL(R)), intermediate reciprocity (LIN(R)), and weak reciprocity (TOT(R)) (See Figure 1), strong alternative reciprocity (FUL($R^{\vee}$)), intermediate alternative reciprocity (LIN($R^{\vee}$)), and weak alternative reciprocity (TOT($R^{\vee}$)). Among alternative reciprocal interpretations two are linguistically attested: intermediate alternative reciprocity exhibited by sentence (4) and weak alternative reciprocity occurring in sentence (5)(See Figure 2).

So we can define 6 corresponding reciprocal quantifiers of type (1, 2):

$$\mathsf{R_S}(A, R) \iff \forall x, y \in A(x \neq y \Rightarrow R(x, y)).$$
$$\mathsf{R_S}^{\vee}(A, R) \iff \forall x, y \in A(x \neq y \Rightarrow (R(x, y) \vee R(y, x))).$$

$$\mathsf{R_I}(A, R) \iff \forall x, y \in A(x \neq y \Rightarrow \exists \text{ sequence } z_1, \ldots, z_\ell \in A \text{ such that } z_1 = x \wedge R(z_1, z_2) \wedge \ldots \wedge R(z_{\ell-1}, z_\ell) \wedge z_\ell = y)].$$

$$\mathsf{R_I}^{\vee}(A, R) \iff \forall x, y \in A(x \neq y \Rightarrow \exists \text{ sequence } z_1, \ldots, z_\ell \in A \text{ such that } (z_1 = x \wedge (R(z_1, z_2) \vee R(z_2, z_1)) \wedge \ldots \wedge (R(z_{\ell-1}, z_\ell) \vee R(z_\ell, z_{\ell-1})) \wedge z_\ell = y)].$$

$$\mathsf{R_W}(A, R) \iff \forall x \in A \exists y \in A(x \neq y \wedge R(x, y))].$$

$$\mathsf{R_W}^{\vee}(A, R) \iff \forall x \in A \exists y \in A(x \neq y \wedge (R(x, y) \vee R(y, x))].$$

Formulae (6)–(10) give the readings to sentences (1)–(5).

(6) $\mathsf{R_S}(\text{MP}, \text{ Refer-indirectly})$.

(7) $\mathsf{R_I}(\text{Pitcher}, \text{ Sit-next-to})$.

(8) $\mathsf{R_W}(\text{Pirate}, \text{ Staring-at})$.

(9) $\mathsf{R_I}^{\vee}(\text{Stones}, \text{ Arranged-on-top-of})$.

(10) $\mathsf{R_W}^{\vee}(\text{Planks}, \text{ Stack-atop-of})$.

2.1.1 Inferential Dependencies

Dalrymple et al. (1998) noticed also that under certain properties of the relation some of the possible definitions become equivalent. For example, if the relation in question is symmetric, then obviously alternative versions reduce to their "normal" counterparts. If the relation R is transitive, then $FUL(R) = LIN(R)$. See Dalrymple et al. (1998) for more details.

2.1.2 The Strongest Meaning Hypothesis

In an attempt to explain variations in the literal meaning of the reciprocal expressions Dalrymple et al. (1998) proposed the so-called *Strongest Meaning Hypothesis* (SMH). According to this principle, the reading associated with the reciprocal in a given sentence is the strongest available reading which is consistent with relevant information supplied by the context. Sabato and Winter (2005) proposed a considerably simple system in which reciprocal meanings are derived directly from semantic restrictions using SMH, and characterized this derivation process.

2.2 Quantified Reciprocal Sentences

Consider versions of our sentences with quantifiers in antecedents:

(11) At least 4 parliament members refer to each other indirectly.

(12) Most Boston pitchers sat alongside each other.

(13) Some Pirates were staring at each other in surprise.

(14) Most stones are arranged on top of each other.

(15) All planks were stacked atop of each other.

To account for their semantics we define lifts turning quantifiers of type (1, 1) into type (1, 2). For the sake of simplicity we will restrict ourselves to reciprocal sentences with right monotone increasing quantifiers in antecedents. Below defined lifts can be extended to cover also sentences with decreasing and non-monotone quantifiers, for example by following the strategy of bounded composition in Dalrymple et al. (1998) or determiner fitting operator in Avi and Winter (2003).(Compare with collective lifts)

$$\mathsf{Ram}_{\mathsf{S}}(\mathsf{Q})AR \iff \exists X \subseteq A[\mathsf{Q}(A, X) \wedge \forall x, y \in X(x \neq y \Rightarrow R(x, y))].$$

$$\begin{aligned}\mathsf{Ram}_{\mathsf{I}}(\mathsf{Q})AR \iff & \exists X \subseteq A[\mathsf{Q}(A, X) \wedge \forall x, y \in X \\ & (x \neq y \Rightarrow \exists \text{ sequence } z_1, \ldots, z_\ell \in X \text{ such that} \\ & z_1 = x \wedge R(z_1, z_2) \wedge \ldots \wedge R(z_{\ell-1}, z_\ell) \wedge z_\ell = y)].\end{aligned}$$

$$\mathsf{Ram}_{\mathsf{W}}(\mathsf{Q})AR \iff \exists X \subseteq A[\mathsf{Q}(A, X) \wedge \forall x \in X \exists y \in X(x \neq y \wedge R(x, y))].$$

Formulae (16)–(20) (alternative lifts are defined obviously) give the readings to sentences (11)–(15).

(16) $\mathsf{Ram}_{\mathsf{S}}(\mathsf{At\ least}\ 4)$MP Refer-indirectly.

(17) $\mathsf{Ram}_{\mathsf{I}}(\mathsf{Most})$Pitcher Sit-next-to.

(18) $\mathsf{Ram}_{\mathsf{W}}(\mathsf{Some})$Pirate Staring-at.

(19) $\mathsf{Ram}_{\mathsf{I}}{}^{\vee}(\mathsf{Most})$Stones Arranged-on-top-of.

(20) $\mathsf{Ram}_{\mathsf{W}}{}^{\vee}(\mathsf{All})$Planks Stack-atop-of.

Homework 1 *Give a meaning representation for the sentence "Most of the students gave each other measles". Justify your choice using SMH.*

2.3 Different Approach

Alternative approach is to analyze reciprocals as anaphoric noun phrases with the addition of plural semantics. It stems from the work of Heim et al. (1991) and culminates in the paper of Beck (2000). We just give one example of a paraphrase in that spirit.

(21) Mary and Bill saw each other.

(22) Mary and Bill saw the other one among Mary and Bill.

(23) Each of Mary and Bill saw every other one of Mary and Bill.

(24) $\forall x(x \in M\&B \rightarrow \forall y(y \in M\&B \wedge y \neq x \rightarrow Saw(x, y)))$.

The idea is then that the reciprocal denotes a group that contains all the members of the antecedent, minus the individual we are looking at in terms of distribution (Mary must have seen everyone among Mary and Bill minus Mary, Bill everyone minus Bill). The reciprocal incorporates two anaphoric dependencies: one is co-reference with the antecedent, the other is dependence on the variable bound in the distribution over the antecedent.

3 Branching Readings

Consider the following examples:

(25) Some book by every author is referred to in some essay by every critic.

(26) Some relative of each villager and some relative of each townsman hate each other.

In particular, Hintikka (1973) proposed that the interpretation of sentence (26) is as follows:

$$\begin{matrix} \forall x \exists y \\ \forall z \exists w \end{matrix} ((V(x) \wedge T(z)) \Rightarrow (R(x, y) \wedge R(z, w) \wedge H(y, w))).$$

What is equivalent to the following Σ_1^1-sentence:

$$\exists f \exists g \forall x \forall z((V(x) \wedge T(z)) \Rightarrow R(x, f(x)) \wedge R(z, g(z)) \wedge H(f(x), g(z)))).$$

Other examples of branching sentences were given by Jon Barwise (1979).

(27) Most villagers and most townsmen hate each other.

Branching reading of sentence (27) is the following:

$$\begin{matrix} \mathsf{MOST}\ x : V(x) \\ \mathsf{MOST}\ y : T(y) \end{matrix}\ H(x, y).$$

Equivalently,

$$\exists A \exists B[\mathsf{MOST}x(V(x), A(x)) \wedge \mathsf{MOST}y(T(y), B(y)) \wedge \forall x \forall y(A(x) \wedge B(y) \Rightarrow H(x, y))].$$

4 Further Readings

For a discussion of other linguistically relevant polyadic lifts of quantifiers see Westerståhl and Peters (2006, Chap. 10). You might be also interested in the next session of Logic, Language and Reasoning Seminar which will be devoted to empirical evidence on branching quantification in natural language. See http://staff.science.uva.nl/~szymanik/LLR.html for more information.

Homework 2 *Give iterated and cumulative interpretation to the following sentence: "Sixty professors taught seventy courses. " What is the difference?*

References

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