

An introduction to the collective quantification

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“When I come no collections will have to be made.”
(1 Corinthians 16:2)

This handout — as well as other materials and homeworks — may be downloaded from the “Structure for Semantics” course web page: <http://staff.science.uva.nl/~szymanik/structures>

For a survey on plurals and collectivity see [Lønning \(1997\)](#).

1 Basic facts and examples

Plural is a grammatical notion. Collectivity is a semantic notion.

Already [Russell \(1903\)](#) discussed collective quantification as opposed to distributive quantification:

- (1) Bunsen and Kirchoff laid the foundations of spectral theory. (together)
- (2) (But:) Armstrong and Aldrin walked on the moon. (separately)
- (3) The Romans conquered Gaul.
- (4) Six hundred tourists visited the Colloseum.

Not all plural NPs are read collectively:

- (5) Some boys like to sing. ($\text{SOME}_{\text{PL}}(A, B) \iff \text{card}(A \cap B) \geq 2$)
- (6) Some boy like to sing.

Collective properties: to meet in secret, to be friends, to love each other, to be twelve in number, to gather, to surround, to lift, ...

2 Higher-order approaches

- (7) Five people lifted the table.
- (8) $\exists^{=5}x[\text{People}(x) \wedge \text{Lift}(x)]$.
- (9) $\exists X[\text{Card}(X) = 5 \wedge X \subseteq \text{People} \wedge \text{Lift}(X)]$.
- (10) Some students played poker together.
- (11) $\exists X[X \subseteq \text{Students} \wedge \text{Play}(X)]$.
- (12) All combinations of cards are losing in some situations.
- (13) $\forall X[X \subseteq \text{Cards} \rightarrow \text{Lose}(X)]$.

All the examples above can be described in terms of the uniform procedure of turning a determiner of type $((et)((et)t))$ into a determiner of type $((et)((et)t)t)$ by means of the type-shifting operator called *existential modifier*, $(\cdot)^{EM}$.

Definition 1 (van der Does (1992)) Fix a universe of discourse U and take any $X \subseteq U$, and $Y \subseteq \mathcal{P}(U)$. Define the existential lift Q^{EM} of a quantifier Q in the following way:

$$Q^{EM}(X, Y) \text{ is true} \iff \exists Z \subseteq X [Q(X, Z) \wedge Z \in Y].$$

2.1 Playing with monotonicity

Consider the following sentence:

- (14) Exactly 5 students drank a whole glass of beer together.

$$(\exists^{=5})^{EM}(\text{Student}, \text{Drink}) \iff \exists A \subseteq \text{Student} [\text{card}(A) = 5 \wedge \text{Drink}(A)].$$

It fails to take into account the total number of students who drank a glass of beer. Then consider:

Definition 2 (van der Does (1992)) Let U be a universe and $X \subseteq U$, $Y \subseteq \mathcal{P}(U)$. We define neutral modifier, $(\cdot)^N$, as follows:

$$Q^N(X, Y) \iff Q(X, \bigcup(Y \cap \mathcal{P}(X))).$$

We now can express the following:

$$(\exists^{=5})^N(\text{Student}, \text{Drink}) \iff \text{card}(\{x : \exists A \subseteq \text{Student}[x \in A \wedge \text{Drink}(A)]\}) = 5.$$

This analysis requires that the total number of students in sets of students that drank a glass of beer together is five. However, it does not require that there was a set of five students who drank a whole glass of beer together.

To overcome the problem let us combine these two modifiers:

Definition 3 (Winter (2001)) The $(\cdot)^{dfit}$ operator turns a determiner of type $((et)((et)t))$ to a determiner of type $((et)t)((et)t)$, i.e., for all $X, Y \subseteq \mathcal{P}(U)$ we have that

$$Q^{dfit}(X, Y) \text{ is true}$$

$$\iff$$

$$Q[\cup X, \cup(X \cap Y)] \wedge [X \cap Y = \emptyset \vee \exists W \in X \cap Y \wedge Q(\cup X, W)].$$

$$(\exists^{=5})^{dfit}(\text{Student}, \text{Drink}) \iff \text{card}(\{x \in A : A \subseteq \text{Student} \wedge \text{Drink}(A)\}) = 5 \wedge \\ \wedge \exists W \subseteq \text{Student}[\text{Drink}(W) \wedge \text{card}(W) = 5].$$

Homework 1 Now consider the following 2 sentences, downward and upward monotone, respectively:

(15) Less than 5 students smiled.

(16) More than 5 students smiled.

What interpretation could you assign to (15) using existential modifier and to (16) using neutral modifier? What are the problems with such interpretations? How determiner fitting operator solves them?

See *Avi and Winter (2003)* for more on the interplay between collective quantification and monotonicity.

2.2 Generalizing GQs

Definition 4 A second order structure of type is a structure of the form (M, P_1, \dots, P_w) , where $P_i \subseteq \mathcal{P}(M^{\ell_i}) \times \dots \times \mathcal{P}(M^{\ell_{r_i}})$.

Definition 5 A second-order generalized quantifier Q is a class of second-order structures such that Q is closed under isomorphisms.

The following examples show that second-order generalized quantifiers are a natural extension from the first-order case.

$$\begin{aligned}\exists^2 &= \{(M, P) \mid P \subseteq \mathcal{P}(M) \text{ and } P \neq \emptyset\}. \\ \text{EVEN} &= \{(M, P) \mid P \subseteq \mathcal{P}(M) \text{ and } \text{card}(P) \text{ is even}\}. \\ \text{EVEN}' &= \{(M, P) \mid P \subseteq \mathcal{P}(M) \text{ and } \forall X \in P(\text{card}(X) \text{ is even})\}. \\ \text{MOST} &= \{(M, P, S) \mid P, S \subseteq \mathcal{P}(M) \text{ and } \text{card}(P \cap S) > \text{card}(P \setminus S)\}.\end{aligned}$$

More on this approach can be found in [Kontinen and Szymanik \(2008\)](#). Studying complexity of second-order GQs we also argue that second-order definable quantifiers are probably not expressive enough to formalize all collective quantification in natural language. As all collective modifiers are definable in second-order logic it might be taken as a good reason to consider different approaches to collective quantification.

3 Many-sorted approaches

Semantics for collectives should obey the following intuitive principles:

Atomicity Each collection is constituted by all the individuals it contains.

Completeness Collections may be combined into new collections.

Atoms Individuals are collections consisting of only a single member.

[Link \(1983\)](#) has the idea of replacing the domain of discourse, which consists of entities, with the structure of a complete atomic join semilattice (CAJS). The author focuses on the cumulative properties of mass nouns (“any sum of parts which are water is water”) and observes that the same approach

can be applied to cover plural nouns. The idea is to enrich the structure of models to account for cumulative references. The main advantage of this algebraic perspective is that it unifies the view on collective predication and predication involving mass nouns. Below we give some details.

Definition 6 (X, \leq) is partially ordered iff \leq on X is reflexive, transitive, and anti-symmetric.

Definition 7 A partially ordered set (X, \leq) is a join-semilattice if any two elements $a, b \in X$ have a supremum (sum, join) $c = a \cup b$, i. e. $a \leq c$ and $b \leq c$ and for all d , if $a \leq d$ and $b \leq d$ then $c \leq d$.

Definition 8 If for every non-empty $A \subseteq X$ $\bigcup A$ is defined in (X, \leq) then we say that (X, \leq) is complete.

Definition 9 An element x of (X, \leq) is an atom, iff (X, \leq) has the smallest element, called 0 , such that $0 \leq x$ and there exists no element y such that $0 \leq y \leq x$.

Definition 10 (X, \leq) is atomic if every non-zero element is join of atoms.

Definition 11 (CAJS-models) Let X be a non-empty set. A CAJS model M is of the form $(\mathcal{P}^+(X), AT(X), (\cdot)^M)$, where the set $AT(X) = \{\{d\} : d \in X\}$ contains the atoms of $\mathcal{P}^+(X)$. The interpretation function, $(\cdot)^M$, assigns a collection $c^M \in \mathcal{P}^+(X)$ to every collective constant c and an atom $a^M \in AT(X)$ to any individual constant a . Moreover, it also assigns to each n -ary collective predicate an n -ary collective relation $\subseteq \mathcal{P}^+(X)^n$ and to each first-order predicate relation among atoms.

CAJS-models come with formal languages, like many sorted first-order logic. The first sort, E , corresponds to entities (elements of $AT(X)$ of type (e)) and the second one, O , to collections (elements of $\mathcal{P}^+(X)$ of type (o)). Such logics usually contain pluralization operator $(\cdot)^*$ turning objects of type (e) into type (o) . Also some operations on objects of type (o) corresponding to \cup are allowed, like \oplus . For more details read chapter 4 in [Lønning \(1997\)](#).

References

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