

Quantifiers & Cognition

Lecture 4: Quantifiers and Monotonicity

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Outline

Monotone Quantifiers

Monotonicity and reasoning

Monotonicity and Negativity Confound

Sentence-picture Verification Models

Experimental hypotheses based on complexity

Experiments

Results

Discussion

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Upward Quantifiers

Definition

A quantifier Q_M of type (n_1, \dots, n_k) is *monotone increasing in the i 'th argument* (upward monotone) iff the following holds:

If $Q_M[R_1, \dots, R_k]$ and $R_i \subseteq R'_i \subseteq M^{n_i}$, then

$Q_M[R_1, \dots, R_{i-1}, R'_i, R_{i+1}, \dots, R_k]$, where $1 \leq i < k$.

Downward Quantifiers

Definition

A quantifier Q_M of type (n_1, \dots, n_k) is *monotone decreasing in the i 'th argument* (downward monotone) iff the following holds:

If $Q_M[R_1, \dots, R_k]$ and $R'_i \subseteq R_i \subseteq M^{n_i}$, then

$Q_M[R_1, \dots, R_{i-1}, R'_i, R_{i+1}, \dots, R_k]$, where $1 \leq i < k$.

For type (1,1)

\uparrow MON $Q_M[A, B]$ and $A \subseteq A' \subseteq M$ then $Q_M[A', B]$.

\downarrow MON $Q_M[A, B]$ and $A' \subseteq A \subseteq M$ then $Q_M[A', B]$.

MON \uparrow $Q_M[A, B]$ and $B \subseteq B' \subseteq M$ then $Q_M[A, B']$.

MON \downarrow $Q_M[A, B]$ and $B' \subseteq B \subseteq M$ then $Q_M[A, B']$.

Monotone quantifiers

Definition

We say that a quantifier is *monotone* if it is monotone decreasing or increasing in any of its arguments. Otherwise, we call it *non-monotone*.

Examples

Example (Square of opposition)

some is \uparrow *MON* \uparrow

no is \downarrow *MON* \downarrow

not all is \uparrow *MON* \downarrow

all is \downarrow *MON* \uparrow

Example

most is \sim *MON* \uparrow

even is \sim *MON* \sim

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Reasoning patterns

Definition

Q is upward monotone if $X \subseteq Y$, then $Q(X)$ entails $Q(Y)$.

1. Every boy runs fast.
2. Every boy runs.

Definition

Q is downward monotone if $Y \subseteq X$, then $Q(X)$ entails $Q(Y)$.

1. No boy runs.
2. No boy runs fast.

Monotonicity calculus

- ▶ Logic rendering many valid arguments.
- ▶ Including syllogistic.
- ▶ Pivoting on monotonicity, e.g.,

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Rule 1	Rule 2	Example 1	Example 2
$\alpha \implies \beta$	$\beta \implies \alpha$	$all(A, B)$	$all(C, B)$
$\dots \alpha^+ \dots$	$\dots \alpha^- \dots$	$some(A^+, C)$	$no(B^-, A)$
$\dots \beta^+ \dots$	$\dots \beta^- \dots$	$some(B^+, C)$	$no(C^-, A)$

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Processing model: minimal proof



Geurts, Reasoning with quantifiers, Cognition, 2003

Monotonicity profiles determine difficulty

1. Some of the sopranos sang with more than three of the tenors.
2. None of the sopranos sang with fewer than three of the tenors.
3. Some of the sopranos sang with fewer than three of the tenors.

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$$\begin{array}{l} Q_1 A \text{ played against } Q_2 B \\ \text{All } B \text{ were } C. \\ \hline Q_1 A \text{ played against } Q_2 C \end{array}$$

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3. Some of the sopranos sang with fewer than three of the tenors.

$$\frac{Q_1 A \text{ played against } Q_2 B}{\text{All } B \text{ were } C.}$$

$$Q_1 A \text{ played against } Q_2 C$$

$$\begin{array}{l} \uparrow Q_1 \uparrow Q_2 < \begin{array}{l} \uparrow Q_1 \downarrow Q_2 \\ \downarrow Q_1 \uparrow Q_2 \\ \downarrow Q_1 \downarrow Q_2 \end{array} \end{array} \quad \begin{array}{l} \uparrow Q_1 \uparrow Q_2 < \downarrow Q_1 \downarrow Q_2 < \begin{array}{l} \uparrow Q_1 \downarrow Q_2 \\ \downarrow Q_1 \uparrow Q_2 \end{array} \end{array}$$



Geurts and Van der Slik, Monotonicity and Processing Load, Journal of Semantics, 2005

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Negativity is marked, not only linguistically

Example

How tall are you? **but not** How short are you?

Negativity is marked, not only linguistically

Example

How tall are you? **but not** How short are you?

Example (Squirrel Monkeys)

1. If everything is black, choose the biggest object.
2. If everything is white, choose the smallest object.

Once trained, monkey were consistently faster in task 1.



McGonigle and Chalmers, The ontology of order, 1996

Just and Carpenter 1971

Observation

Processing time of negative quantifiers is greater than processing time of affirmative quantifiers.



Just & Carpenter, Comprehension of negation with quantification, Journal of Verbal Learning and Verbal Behavior, 1971

3 kinds of sentences

1. Syntactic negatives with a particle:
 - ▶ The dots are red.
 - ▶ The dots aren't red.

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 - ▶ Few of the dots are red.

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2. Syntactic negatives without particle:
 - ▶ Many of the dots are red.
 - ▶ Few of the dots are red.
3. Semantic negatives:
 - ▶ A majority of the dots are red.
 - ▶ A minority of the dots are red.

Only some pairs contrasted w.r.t. monotonicity:

1. All of the dots are red.
2. None of the dots are red.

Most of the material was based on negativity vs. affirmativity.

Affirmativity and monotonicity?

- ▶ Monotonicity is a semantic property of quantifiers;
- ▶ Degree of affirmativity is a linguistic concept, e.g.,:

Affirmativity and monotonicity?

- ▶ Monotonicity is a semantic property of quantifiers;
- ▶ Degree of affirmativity is a linguistic concept, e.g.,:
 - ▶ tag test;
 - ▶ licensing NPIs.

Example

1. Few children are dirty, are they?
2. Few children believe that any more.
3. *A few children are dirty, are they?
4. *A few children believe that any more.

A partial dissociation

Observation

Dissociation between downward monotonicity and negativity.

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1. At most half of the children believe that, don't they?
2. Not many children believe that, do they?

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Example

1. At most half of the children believe that, don't they?
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Conclusion

Downward monotone quantifiers fall into two classes: affirmatives and negatives.

Negativity/affirmativity and processing

Claim

Differences in processing caused by negativity/affirmativity.

Negativity/affirmativity and processing

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Differences in processing caused by negativity/affirmativity.

Observation

Affirmatives focus on reference set and negatives focus on complement set.

Example

1. Q of the football fans went to the match. They ...
2. A few of the football fans went to the match.
They cheered as their team scored.
3. Few of the football fans went to the match.
They stayed at home instead.



Moxey & Sanford, Quantifiers and focus, *Journal of Semantics*, 1987

The confound

Question

Are effects reported by JC'71 due to monotonicity or negativity?

The confound

Question

Are effects reported by JC'71 due to monotonicity or negativity?

- ▶ It's common to mix up monotonicity and negativity.
- ▶ Our aim is to clearly separate those two:
- ▶ Focus on monotonicity.

Polish has no negativity degrees

- ▶ Polish - the language of the experiment.
- ▶ It has no negativity degrees.
- ▶ No dissociation monotonicity/negativity.
- ▶ Multiple negations allowed.

Example

Czy to możliwe, że nikt nigdy nikogo nigdzie nie widział?

Is it possible that no one never no one nowhere didn't see?

“Is it possible that no one has ever seen anyone anywhere?”

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Comparison model

4 stage processing of the comparison model:

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1. sentence encoding,

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Component latencies are additive.

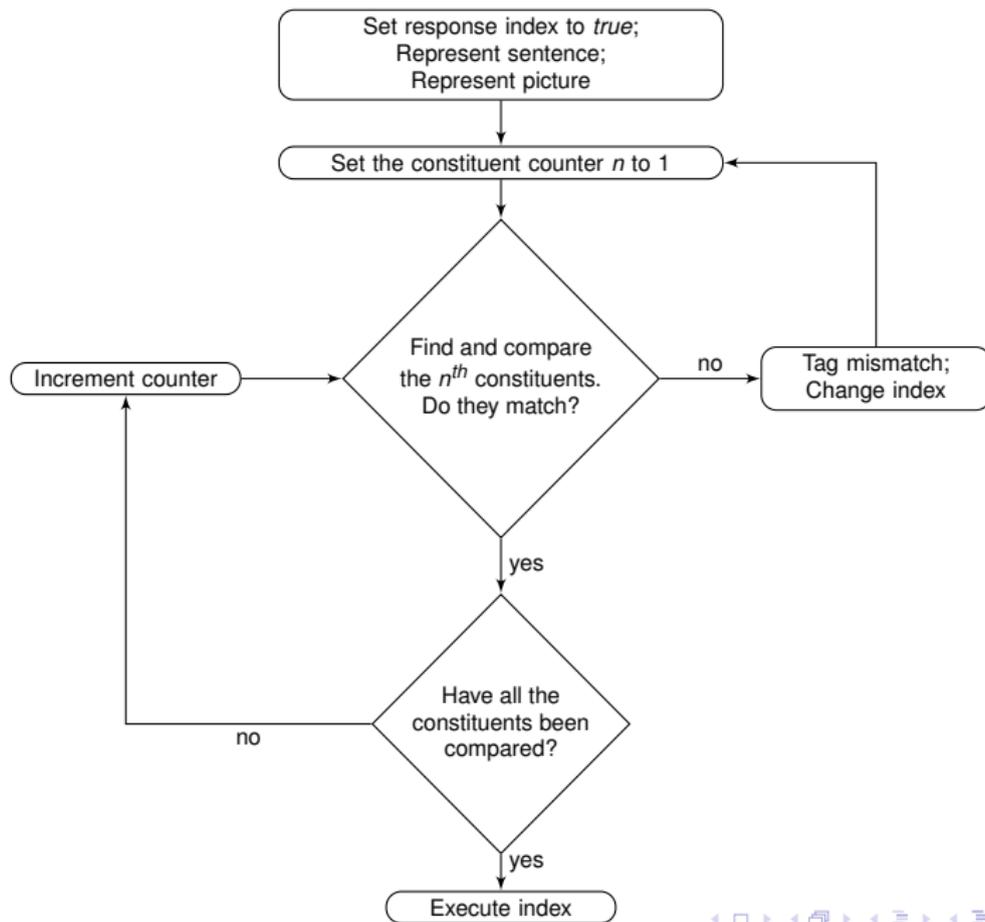


Clark & Chase, On the process of comparing sentences against pictures,
Cognitive Psychology, 1972

Example: The dots are (not) white.

True affirmative		False affirmative	
Comparison: <i>Aff</i> (white, dots) <i>Aff</i> (white, dots) + +	Index <i>true</i>	Comparison: <i>Aff</i> (white, dots) <i>Aff</i> (black, dots) - + + +	Index <i>false</i> <i>false</i>
2 comparisons		3 comparisons	
True negative		False negative	
Comparison: <i>Neg</i> (white, dots) <i>Aff</i> (white, dots) - + + +	Index <i>false</i>	Comparison: <i>Neg</i> (white, dots) <i>Aff</i> (black, dots) - + + +	Index <i>false</i> <i>true</i>
4 comparisons		5 comparisons	

Computational model



Pros & Cons of the comparison model

Pros

- ▶ *Explains variations w.r.t. negativity:*

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- ▶ *How to cover quantifiers?*

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Conclusion

- ▶ *How to cover quantifiers?*
- ▶ *It says very little about monotonicity.*

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Conclusion

- ▶ *How to cover quantifiers?*
- ▶ *It says very little about monotonicity.*
- ▶ *Arbitrary psychological representation.*

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Pros

- ▶ *Explains variations w.r.t. negativity:*
 - ▶ *negatives harder than affirmatives;*
 - ▶ *true affirmatives easier than false ones;*
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Conclusion

- ▶ *How to cover quantifiers?*
- ▶ *It says very little about monotonicity.*
- ▶ *Arbitrary psychological representation.*
- ▶ *Little insight into the actual computational process.*

Question

What can we predict about monotonicity from c.c.?

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Design

- ▶ 4 different quantifiers:
 - ▶ Cardinal quantifiers (“more than 7”, “fewer than 8”);
 - ▶ Proportional (“more than half”, “fewer than half”).

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- ▶ True vs. false.

Design

- ▶ 4 different quantifiers:
 - ▶ Cardinal quantifiers (“more than 7”, “fewer than 8”);
 - ▶ Proportional (“more than half”, “fewer than half”).
- ▶ Upward monotone vs. downward monotone.
- ▶ True vs. false.
- ▶ Subjects were timed when asked to decide if true.

Predictions

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2. Complexity influenced by (monotonicity \times truth-value):
 - ▶ In the case of the cardinal sentences,
 - ▶ but not the proportional sentences.

Predictions in details

1. “More than 7”: `true > false (8>7)`.

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1. “More than 7”: true > false ($8 > 7$).
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Predictions in details

1. “More than 7”: true $>$ false ($8 > 7$).
2. “Fewer than 8”: true $<$ false ($7 < 8$).
3. On average “fewer than 8” $>$ “more than 7”.
4. No difference between proportional quantifiers.
5. Proportional quantifiers $>$ cardinal quantifiers.

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2 experiments

- ▶ 2 independent experiments.
- ▶ Same results.
- ▶ We report only on the first one.

Participants

- ▶ 30 native Polish-speaking adults (15 female).
- ▶ Volunteers: undergraduates from the University of Warsaw.
- ▶ The mean age: 21.30 years (SD = 2.54).
- ▶ Each participant tested individually.

Materials

16 grammatically simple propositions in Polish, like:

1. More than 7 cars are blue.
2. Fewer than 8 cars are yellow.
3. More than half of the cars are red.
4. Fewer than half of the cars are black.

Materials continued

More than half of the cars are yellow.



An example of a stimulus used in the first study

Procedure

- ▶ Each quantifier was presented in 10 trials.
- ▶ The sentence true in the picture in half of the trials.
- ▶ **Quantity of target items near the criterion of validation.**
- ▶ Practice session followed by the experimental session.
- ▶ Each quantifier problem was given one 15.5 s event.
- ▶ Subjects were asked to decide the truth-value.

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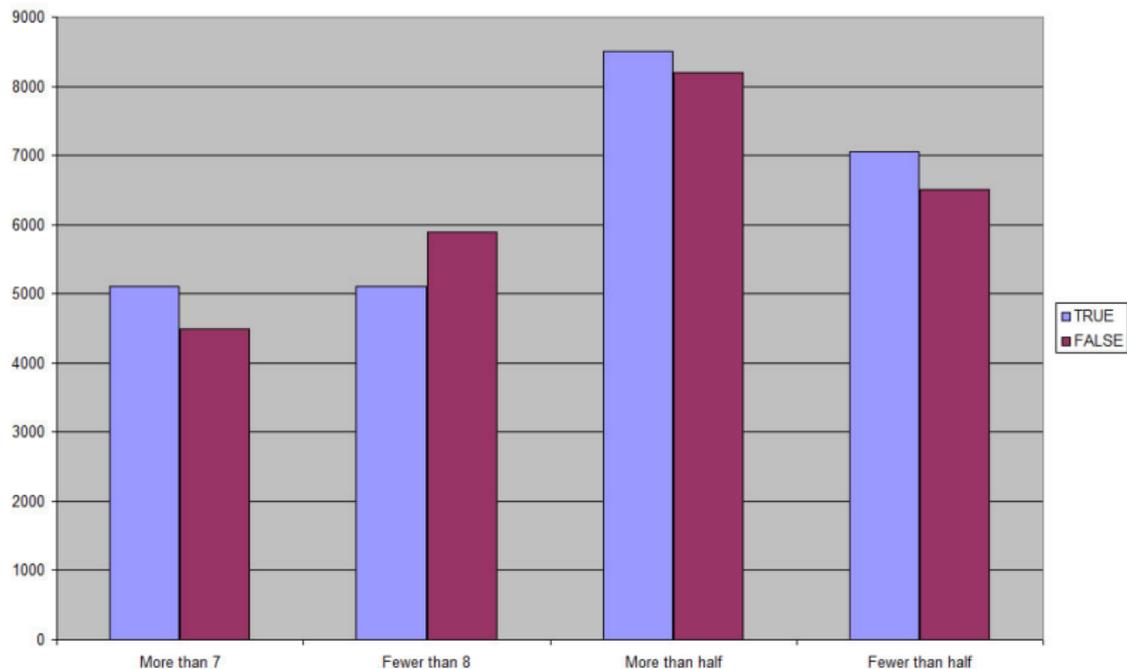
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Raw data

Quantifier		M	SD
More than 7	True	5073.36	1244
	False	4526.36	1299
	Overall	4799.87	1157
Fewer than 8	True	5047.80	1693
	False	5872.13	1795
	Overall	5459.97	1510
More than half	True	7431.12	2076
	False	7195.13	2342
	Overall	7313.42	2016
Fewer than half	True	7064.80	2433
	False	6539.67	2092
	Overall	6802.23	2098

Interaction effect



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cf. Just and Carpenter, 1971

- ▶ Difference between cardinal quantifiers.
- ▶ No difference between proportional quantifiers.

cf. Just and Carpenter, 1971

- ▶ Difference between cardinal quantifiers.
- ▶ No difference between proportional quantifiers.
- ▶ Differences in JC'71 is due to negativity.
- ▶ Vague quantifiers of JC'71 trigger different strategies.

False/True interaction

- ▶ $RT(\exists_T^{\geq 7}) > RT(\exists_F^{\geq 7})?$ 👍
- ▶ $RT(\exists_T^{\leq 8}) < RT(\exists_F^{\leq 8})$. 👍
- ▶ No difference in proportional quantifiers. 👍
- ▶ Complexity hypothesis. 👍

Model's plausibility

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- ▶ Complexity hypothesis derived from the minimal automata.
- ▶ Proportional quantifiers > numerical quantifiers.
- ▶ Monotonicity accounts for 45% of the variance.
- ▶ Complexity for over 90%.
- ▶ Complexity explains more than monotonicity.

Big question

Question

How are reasoning and verification related?

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Hypothesis

Via a comprehension model?