Structures for Semantics: new assignment

[You may e-mail your work until May 8 to r.a.m.vanrooij@uva.nl, or hand it in at that date. In case you have any questions about the exercises, please contact me (Robert van Rooij, room 213, phone 525-4551, r.a.m.vanrooij@uva.nl)]

1 Questions and Aboutness

According to Groenendijk & Stokhof's partition semantics, Q_1 (or Q_2) always either stands for a partition or for the corresponding equivalence relation between worlds. We denote Q_1^R if we think of Q as an equivalence relation, and Q_1^P if we think of Q as a partition. Thinking of questions in terms of equivalence relations we can say that Q_1^R entails Q_2^R, $Q_1^R \models Q_2^R$, iff $Q_1^R \subseteq Q_2^R$.

(a) Show that the above notion of entailment is equivalent with the following condition: $\forall q \in Q_1^P : \exists q' \in Q_2^P : q \subseteq q'$.

We also will use P and P' either as sentences or the propositions they denote, hoping that this will never lead to confusion. Let us now define that P is **about** Q^R iff whenever $\langle w, v\rangle \in Q^R$, then w and v give P the same truth value.

(b) Prove that this definition is equivalent to the following definition:
P is *about* Q^P (thus, Q thought of as a partition) iff $\forall q \in Q^P$: either $q \subseteq P$ or $q \cap P = \emptyset$.

(c) Is aboutness closed under negation and conjunction? That is, does it hold for all propositions P and P' and for all questions Q that (i) if P is about Q, then $\neg P$ is about Q? and (ii) if P and P' are about Q, then $P \wedge P'$ is about Q? (motivate your answer)

(d) Proposition P is about many questions. For instance, every proposition is about the finest grained question that asks exactly how the world looks like. However, for every proposition P there is also a *coarsest* grained question that P is about (where Q_1^R is at least as coarse grained as Q_2^R iff $Q_2^R \subseteq Q_1^R$.) Which question is this for P? (motivate your answer)

(e) Show that the following facts hold:
- If P_1 is about Q_1 and P_2 is about Q_2, then $(Q_1^R \cap Q_2^R) \subseteq (P_1?^R \cap P_2?^R)$
- For any truth-functional compound $@(P_1, P_2)$: $(P_1?^R \cap P_2?^R) \subseteq @(P_1, P_2)?^R$

If both are true, then we can conclude: For any truth-functional compound $@(P_1, P_2)$: if P_1 is about Q_1 and P_2 is about Q_2, then $(Q_1 \cap Q_2?) \subseteq @(P_1, P_2)?$. As a special case it now follows that if both of P_1, P_2 are entirely about a given subject matter, so is $@(P_1, P_2)$.

Let us assume that questions Q_1^R and Q_2^R are *orthogonal* to one another iff for every two worlds w and u there is a world v such that $\langle w, v\rangle \in Q_1^R$ and $\langle v, u\rangle \in Q_2^R$.

(f) Proof that Q_1^R and Q_2^R are orthogonal iff $\forall q_1 \in Q_1^P : \forall q_2 \in Q_2^P : q_1 \cap q_2 \neq \emptyset$.

(g) Intuitively, we conclude from the Austin conditional 'If you are hungry, there are biscuits in the refrigerator' that there are biscuits in the refrigerator. Motivate why this follows on a material implication analysis of conditionals, if we assume that the yes-no questions ‘Are you hungry?’ and ‘Are there biscuits in the fridge?’ are orthogonal to each other.

2 Only

In ‘Only: Meaning and Implicature’ (van Rooij & Schulz, 2007, see my homepage) it is argued that ‘only’ should be interpreted as follows:

$$only^W(A,P) \quad = \quad \{w \in W | \exists v \in exh^W_{std}(A,P) : w \leq_P v\}$$

The reason why we interpreted ‘only’ not in the same way as exhaustive interpretation was that there seem to be good arguments to assume that a sentence like ‘Only [John]$_F$ passed the examination’ does not entail that John passed the examination (even though I don’t believe this anymore now). But if we want ‘only’ to mean the same as exhaustivity except for the fact that a sentence of the form ‘Only A’ does not entail A, it seems more straightforward to analyze ‘only’ as follows:

$$only^W(A,P) \quad = \quad \{w \in W | \neg\exists v \in [A]^W : v <_P w\}$$

Do you think this would have been a good idea, or does it give rise to some strange predictions? If not, show that this latter definition is equivalent to the former one. If the latter formula gives rise to strange predictions, however, show me one, and explain why the latter rule predicts falsely here.

3 Grice

(a) Suppose that there are only three relevant individuals: {John, Mary, Sue}. Suppose, moreover, that we have all 8 relevantly different worlds that give to predicate ‘*Came*’ a different extension. What are the information states in $Grice(\text{John came}, came)$ and $Grice(\text{John and Mary came}, came)$?

(b) Does the function *Grice* applied to a sentence (with a semantic interpretation) and a question predicate P obey the following constraints?

(i) $Grice(A \wedge B, P) = Grice(A, P) \cap Grice(B, P)$.
(ii) $Grice(A \wedge B, P) = \{X \cap Y : X \in Grice(A, P) \wedge Y \in Grice(B, P)\}$.
(iii) $Grice(A \vee B, P) = Grice(A, P) \cup Grice(B, P)$.
(iv) $Grice(A \vee B, P) = \{X \cup Y : X \in Grice(A, P) \wedge Y \in Grice(B, P)\}$.

Show why, or why not (for each constraint individually). Warning: some of these questions are not easy!

(c) Just like the ordering we make use of in our analysis of exhaustification, also our the ordering used in the function *Grice* can be formulated with respect to a predicate and with respect to a set of alternatives (see, among others, the above paper on ‘only’). By defining the ordering in terms of alternatives, we have assumed that these are closed under conjunction and disjunction. Can you give examples where this assumption was crucial? We have also assumed that the set of alternatives was not closed under negation. Why do you think we didn’t assume that? What would follow if we did?

4 Deriving weak orders

A structure $\langle I, R\rangle$, with R a binary relation on I, is a weak order just in case R is irrreflexive (IR), transitive (TR), and almost connected (AC).

Definition 1 *A weak order is a structure $\langle I, R\rangle$, with R a binary relation on I that satisfies the following conditions:*
(IR) $\forall x : \neg R(x, x)$.
(TR) $\forall x, y, z : (R(x, y) \wedge R(y, z)) \rightarrow R(x, z)$.
(AC) $\forall x, y, z : R(x, y) \rightarrow (R(x, z) \vee R(z, y))$.

Definition 2 *A context structure, M, is a triple $\langle I, C, f\rangle$, where I is a non-empty set of individuals, the set of contexts, C, consists of all finite subsets of I, and P a choice function which assigns to each context $c \in C$ the set of individuals of c that count as P in c.*

4.1 Deriving 'better' from 'best'

Arrow (1959) stated the following principle of choice (C), and the constraints (A1) and (A2) to assure that the choice function behaves in a 'consistent' way:

(C) $\forall c \in C : P(c) \neq \emptyset$.
(A1) If $c \subseteq c'$, then $c \cap P(c') \subseteq P(c)$.
(A2) If $c \subseteq c'$ and $c \cap P(c') \neq \emptyset$, then $P(c) \subseteq P(c')$.

Define $x > y$ as follows: $x >_P y$ iff$_{def}$ $x \in P(\{x, y\}) \wedge y \notin P(\{x, y\})$. Prove that the ordering as defined above gives rise to a *weak order*.

4.2 Deriving 'better' from 'good'

Look again at the context structure M, but now assume the following constraints that assure that the choice function behaves in a 'consistent' way. Consider any two individuals $x, y \in c$ such that $x \in P(c)$, but $y \in (c - P(c))$. Let us now make the following constraints on the behavior of P among different contexts:

NO REVERSAL: $\neg\exists c' \in C : y \in P(c') \wedge x \in (c' - P(c'))$.

UPWARD DIFFERENCE: $\forall c' \in C : c \subseteq c' \rightarrow \exists v, w : v \in P(c') \wedge w \in (c' - P(c'))$.

DOWNWARD DIFFERENCE: $\forall c' \in C : (c' \subseteq c \;\&\; x, y \in c') \rightarrow \exists v, w : v \in P(c') \wedge w \in (c' - P(c'))$.

Define $x > y$ as before: $x >_P y$ iff$_{def}$ $x \in P(\{x, y\}) \wedge y \notin P(\{x, y\})$. Prove that the ordering as defined this way gives rise to a *weak order*.

5 Events

Definition 3 *A Russell event structure $\Sigma_R = \langle E, <, \sim, <_B, < E\rangle$, consists of a non-empty set E of events together with four binary relations 'before' ($<$), 'overlap' ($\sim$), 'begins before' ($<_B$), and 'ends-before ($<_E$) such that:*

1. *$<$ is an interval oder;*
2. *$\sim$ is defined as $e \sim e'$ iff$_{def}$ $e \not< e'$ and $e' \not< e$*
3. *$<_B$, and $<_E$ are defined as follows:*
   $e <_B e'$ *iff$_{def}$* $\exists e''(e'' \sim e \wedge e'' < e')$
   $e <_E e'$ *iff$_{def}$* $\exists e''((e < e'' \wedge e'' \sim e')$.

Show that $<_B$ (and $<_E$, but the proof is the same) is a weak order.