Complexity, meaning, and quantifiers

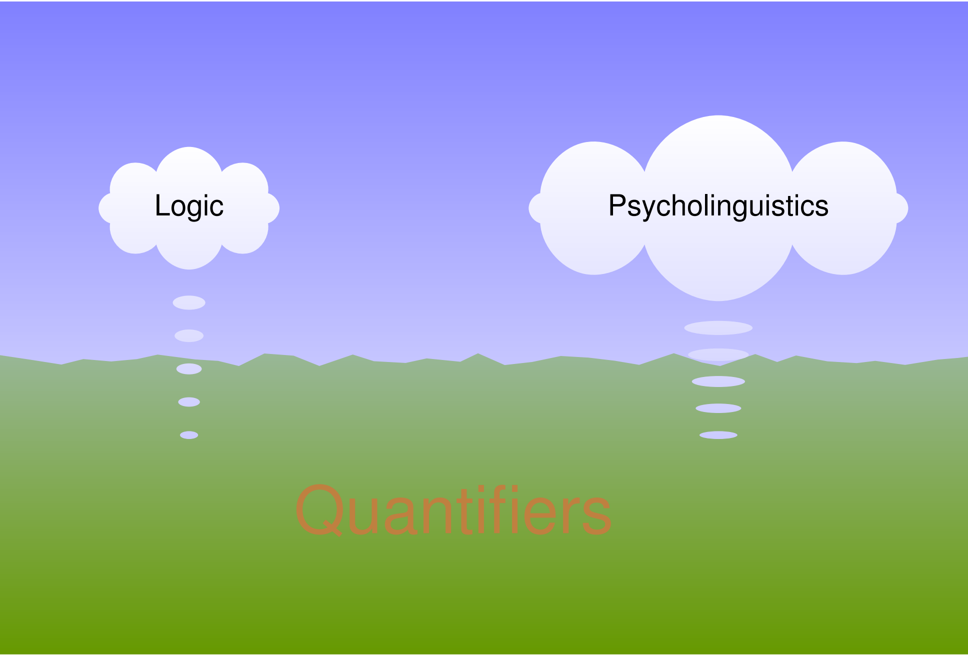
Jakub Szymanik

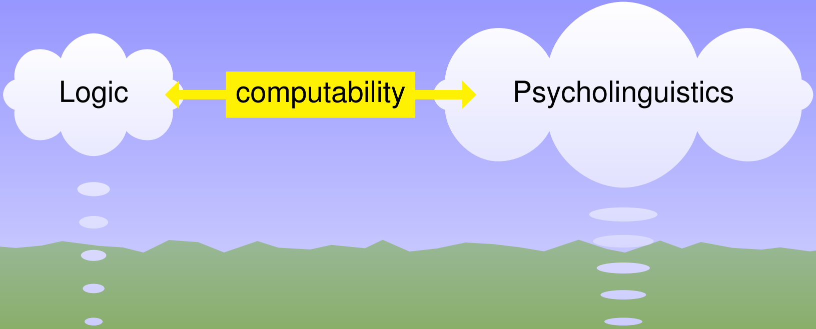
Vague Quantities and Vague Quantifiers
Berlin, December 9, 2010

Quantifiers

Logic

Quantifiers


Logic
Psycholinguistics
Quantifiers



Quantifiers

Plan

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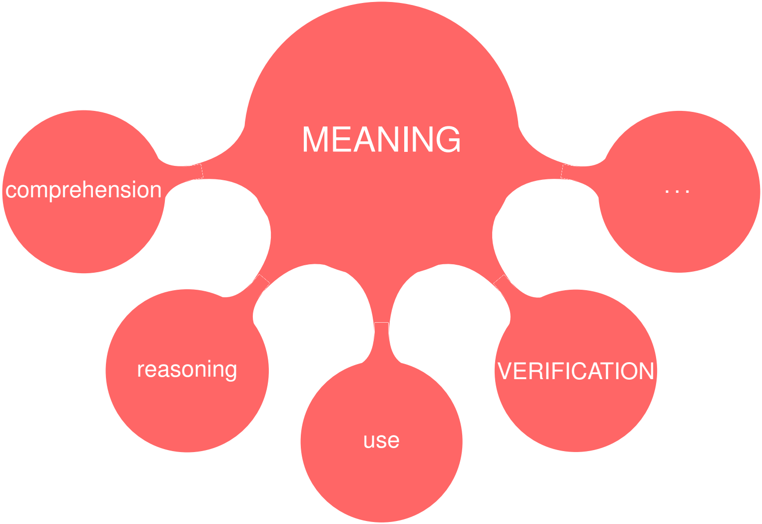
4. Precision costs a lot!
5. So, it might be necessary to approximate.
6. Some evidence on ‘more than half’.

Outline

Aspects of meaning



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  - visual clues;
  - level of precision subjects want to achieve;
  - quantifiers;
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We discussed many examples at that workshop.

Main question

Question

How are different mechanisms related?

Outline

Marr's 3 levels of explanation

1. computational level:
   - problems that a cognitive ability has to overcome

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   - the algorithms that may be used to achieve a solution
3. implementation level:
   - how this is actually done in neural activity

Marr, Vision: a computational investigation into the human representation and processing visual information, 1983

Expression $\overset{\textit{computation}}{\Longrightarrow}$ denotation

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- Ability to understand sentences.
- Capacity of recognizing their truth-values.
- Long-standing tradition.
- Meaning is a procedure for finding extension in a model.
- Adopted often with psychological motivations.

Explicit formulation

Pavel Tichý "Intension in terms of Turing machines", 1969:

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> *[. . .] the fundamental relationship between sentence and procedure is obviously of a semantic nature; namely, the purpose of sentences is to record the outcome of various procedures. Thus e.g. the sentence “The liquid X is an acid” serves to record that the outcome of a definite chemical testing procedure applied to X is positive.*

For what does it mean to understand, i.e. to know the sense of an expression? It does not mean actually to know its denotation but to know how the denotation can be found, how to pinpoint the denotation of the expression among all the objects of the same type. E.g. to know the sense of "taller" does not mean actually to know who is taller than who, but rather to know what to do whenever you want to decide whether a given individual is taller than another one. In other words, it does not mean to know which of the binary relations on the universe is the one conceived by the sense of "taller", but to know a method or procedure by means of which the relation can be identified. (Tichý, 1969)

Psychological motivation

The basic and fundamental psychological point is that, with rare exceptions, in applying a predicate to an object or judging that a relation holds between two or more objects, we do not consider properties or relations as sets. We do not even consider them as somehow simply intensional properties, but we have procedures that compute their values for the object in question. Thus, if someone tells me that an object in the distance is a cow, I have a perceptual and conceptual procedure for making computations on the input data that reach my peripheral sensory system [. . .] Fregean and other accounts scarcely touch this psychological aspect of actually determining application of a specific algorithmic procedure. (Suppes 1982)

Meaning as a collection of procedures

I have defended the thesis that the meaning of a sentence is a procedure or a collection of procedures and that this meaning in its most concrete representation is wholly private and idiosyncratic to each individual. (Suppes 1982)

Question

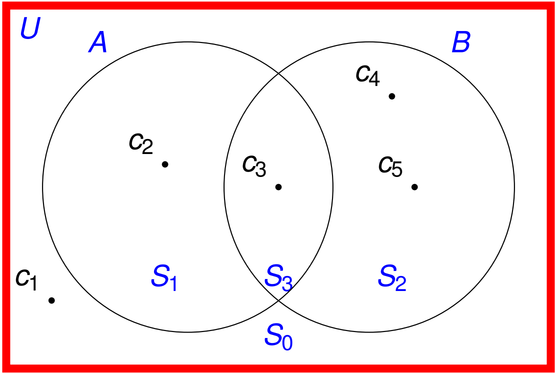
What are we computing in the case of quantifiers?

Outline

What is the semantic assigned to quantifiers?

1. Every poet has low self-esteem.
2. Some dean danced nude on the table.
3. At least 7 grad students prepared presentations.
4. An even number of the students saw a ghost.
5. Most of the students think they are smart.
6. Less than half of the students received good marks.

Illustration



Monadic quantifiers of type (1, 1)

Definition

A monadic generalized quantifier of type (1,1) is a class Q of structures of the form $M = (U, A_1, A_2)$, where $A_1, A_2 \subseteq U$. Additionally, Q is closed under isomorphism.

Examples

$$\text{every} = \{(M, A, B) \mid A, B \subseteq M \text{ and } A \subseteq B\}.$$

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$$\text{most} = \{(M, A, B) \mid A, B \subseteq M \text{ and } \text{card}(A \cap B) > \text{card}(A - B)\}$$

Why are logicians so excited with GQs?

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Everyone knows everyone here.

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Rasmus
Petr
Jakub
Dennis
Frank
Raquel
Ulle
Stephanie
Manfred
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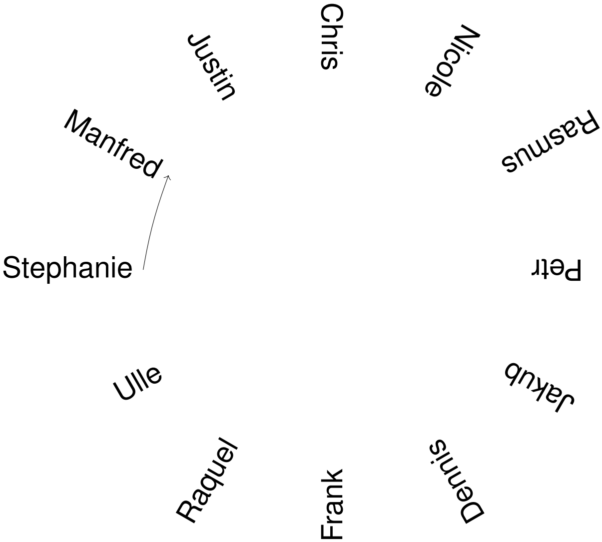

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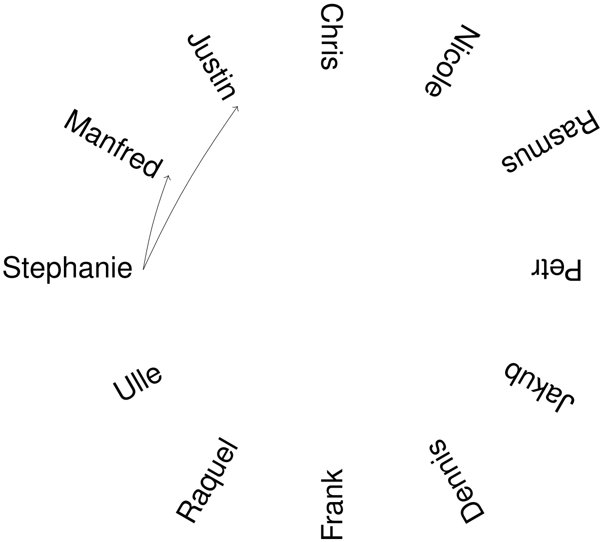

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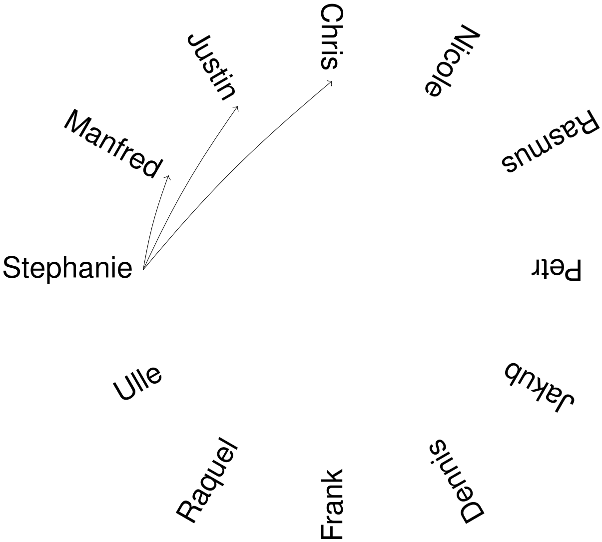

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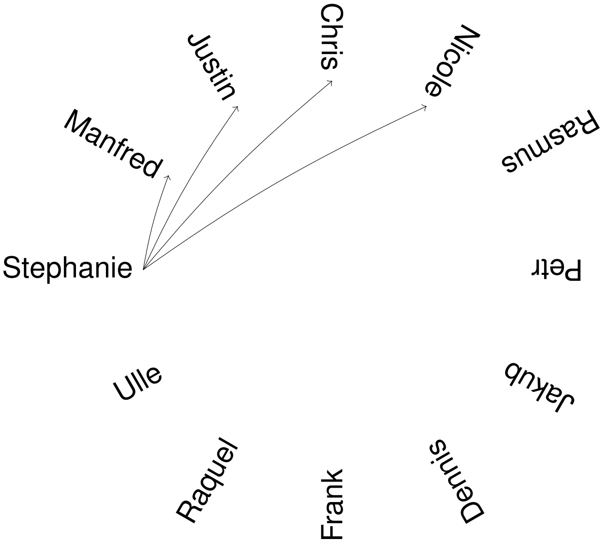

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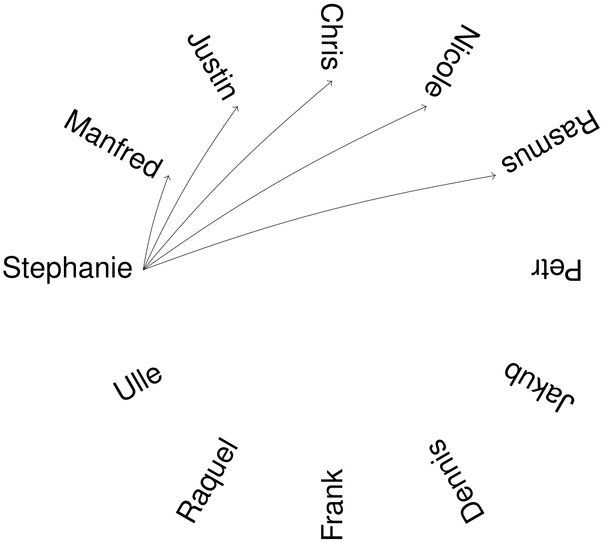

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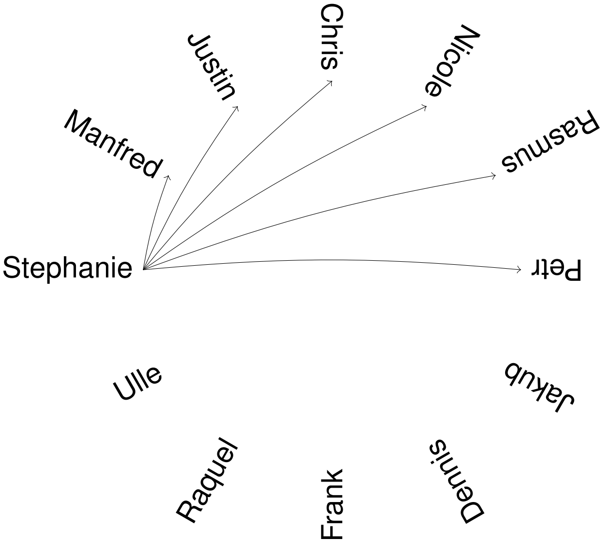

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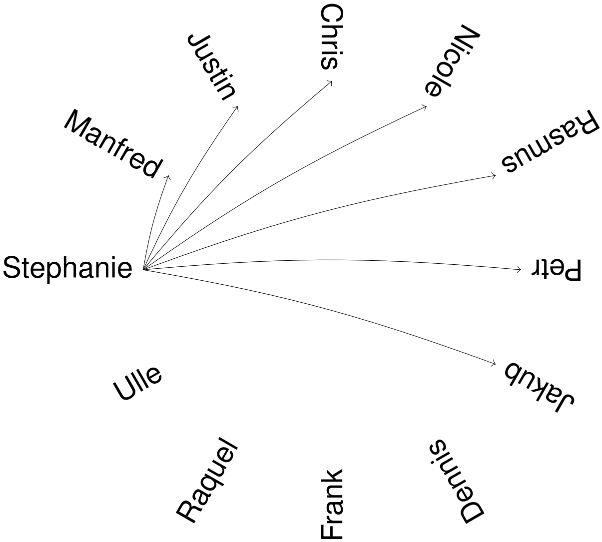

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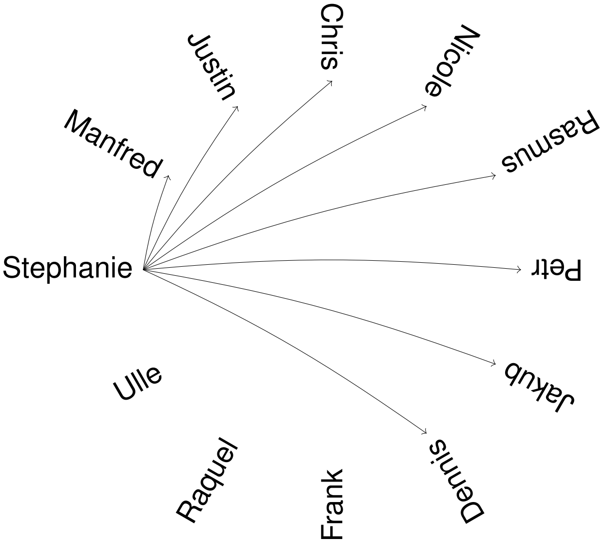

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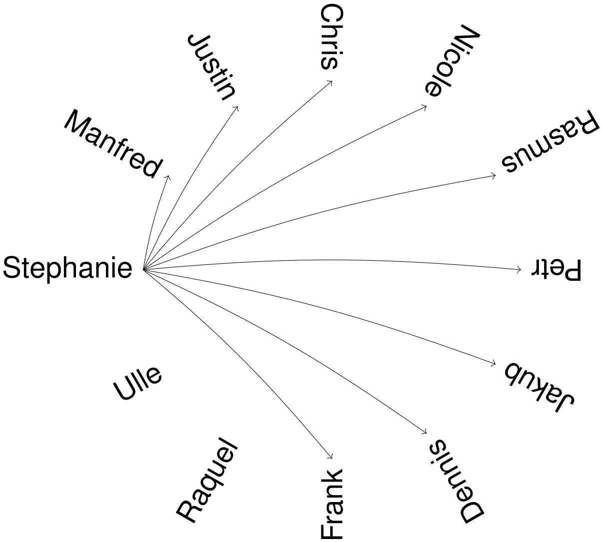

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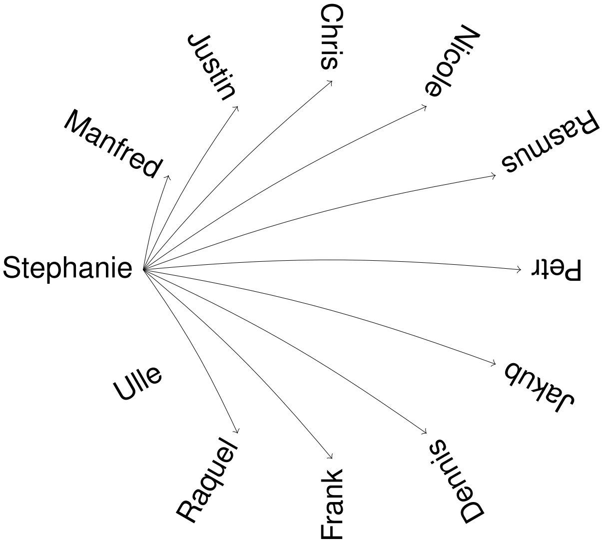

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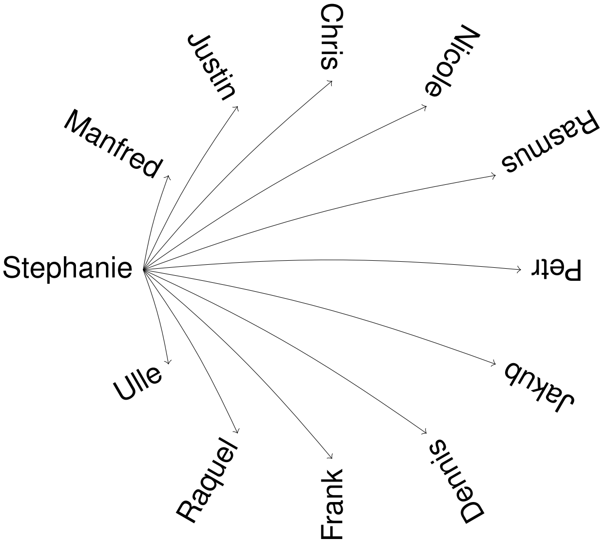

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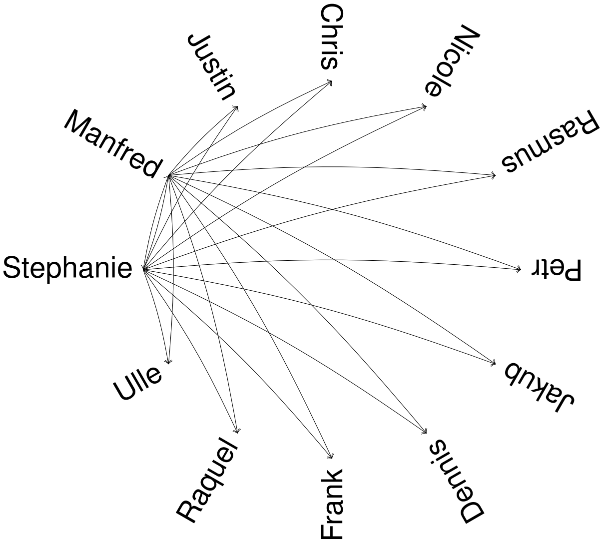

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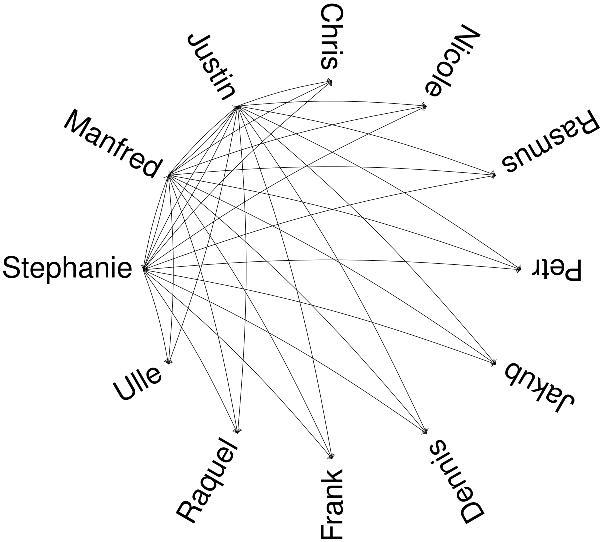

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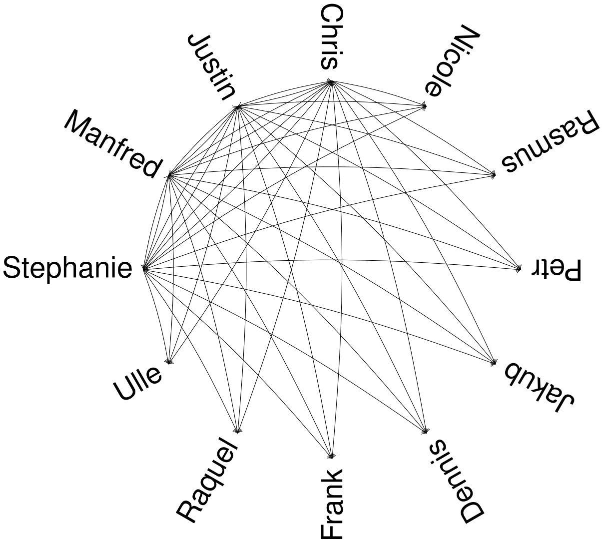

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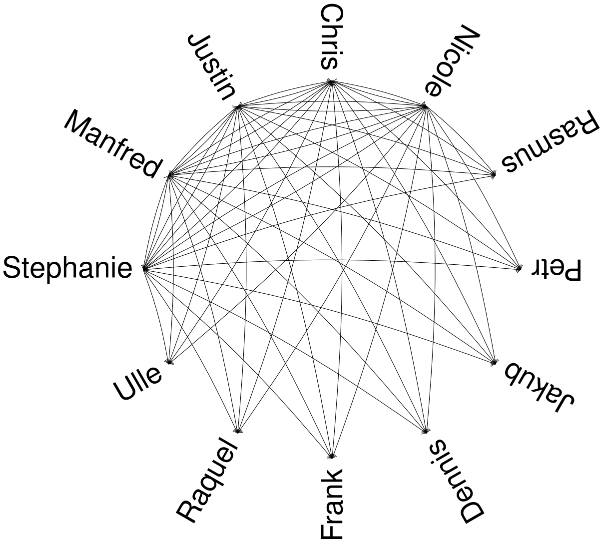

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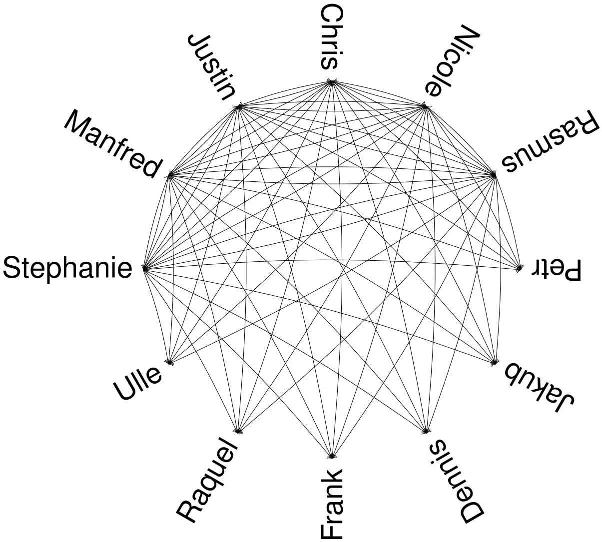

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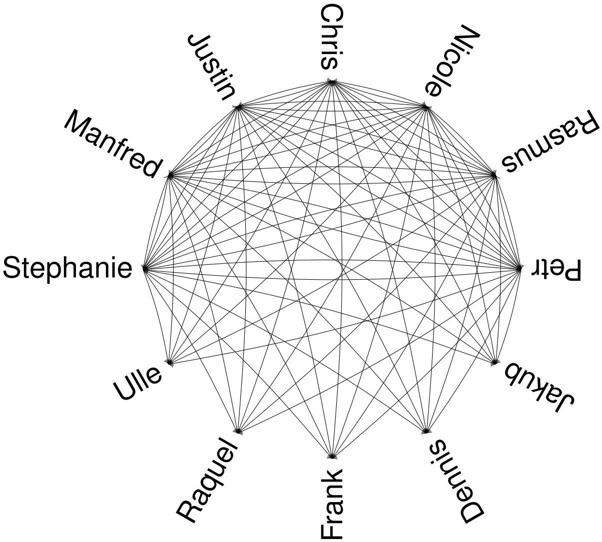

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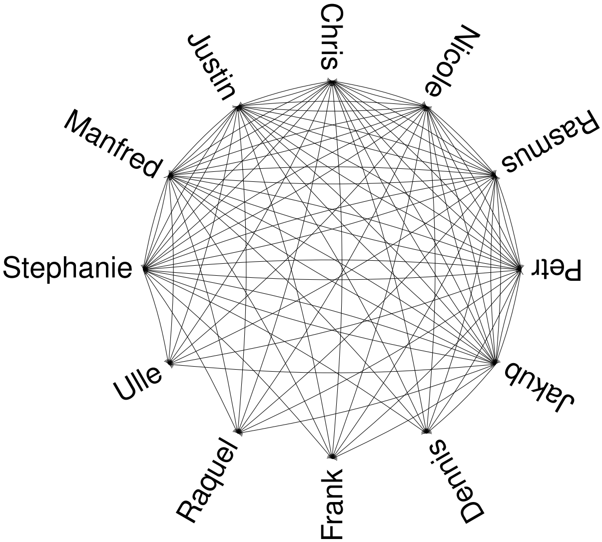

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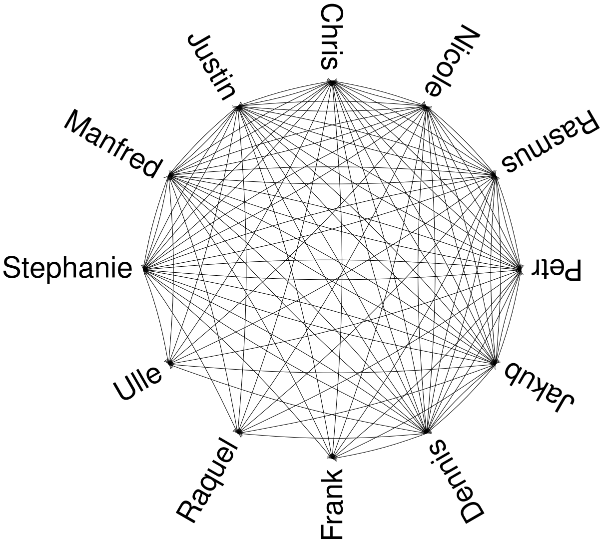

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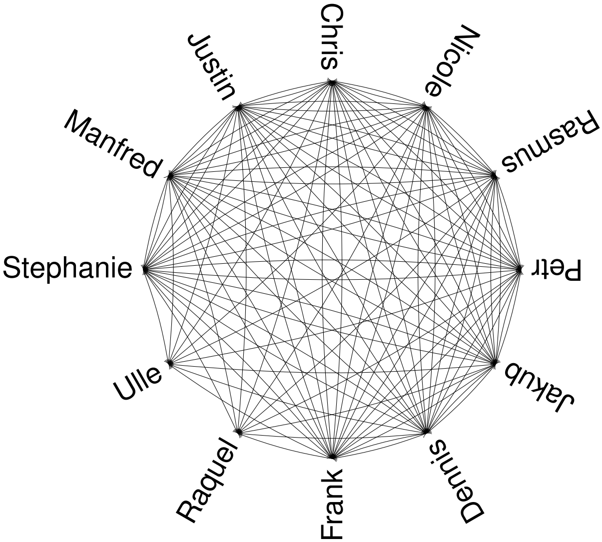

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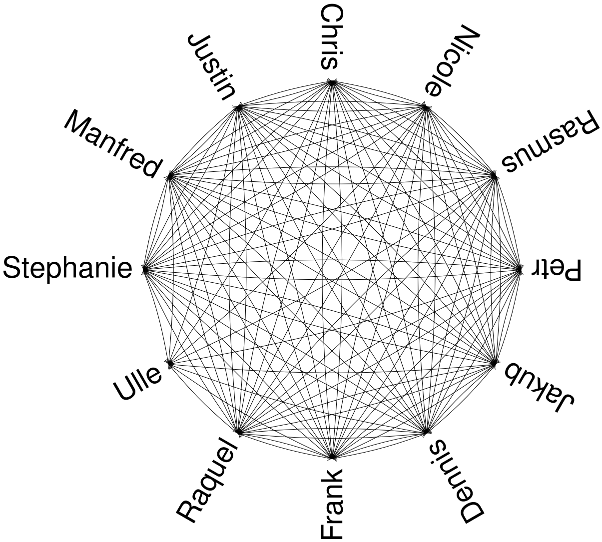

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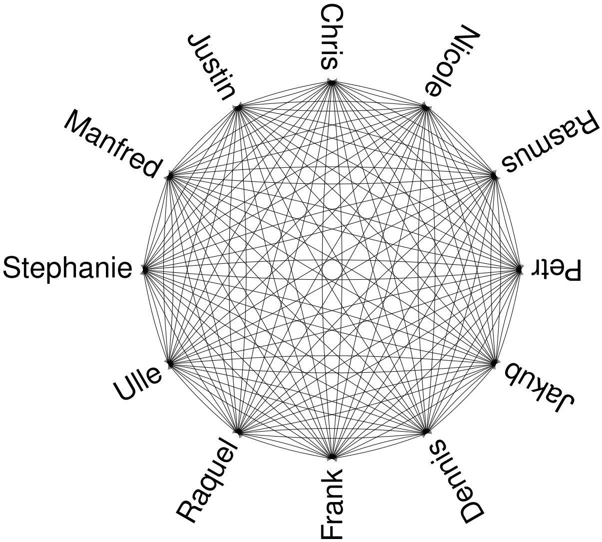


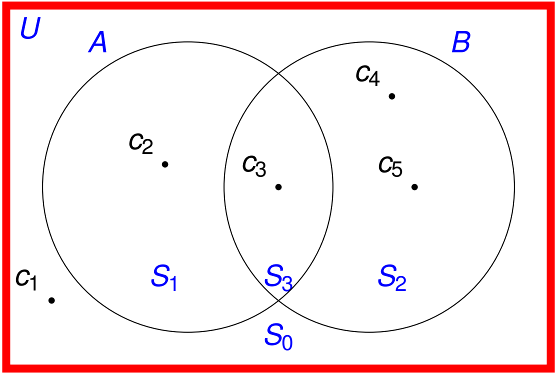
We know what GQs denote. Now, it’s time to see how we compute those denotations.

Outline

Automata perspective

An attractive, but never very central idea in modern semantics has been to regard linguistic expressions as denoting certain "procedures" performed within models for the language. (Van Benthem, 1986)

How do we encode models



This model is uniquely described by $\alpha_M = a_{\bar{A}\bar{B}} a_{A\bar{B}} a_{AB} a_{\bar{A}B} a_{\bar{A}B}$

Step by step

- Restriction to finite models of the form $M = (U, A, B)$.

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- Result: the word $\alpha_M = a_{\bar{A}\bar{B}} a_{A\bar{B}} a_{AB} a_{\bar{A}B} a_{\bar{A}B}$.
- α_M describes the model in which: $c_1 \in \bar{A}\bar{B}, c_2 \in A\bar{B} c_3 \in AB, c_4 \in \bar{A}B, c_5 \in \bar{A}B$.

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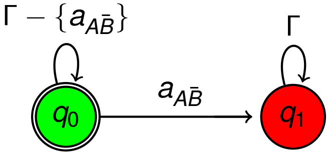
- The class Q is represented by the set of words describing all elements of the class.

Aristotelian quantifiers

“all”, “some”, “no”, and “not all”

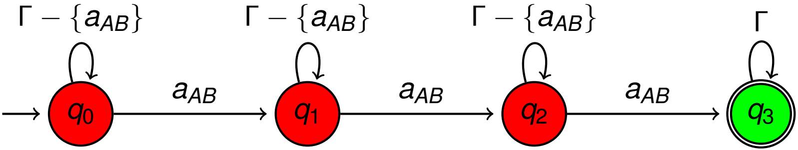


Finite automaton recognizing L_{All}

$$L_{\text{All}} = \{\alpha \in \Gamma^* : \#a_{A\bar{B}}(\alpha) = 0\}$$

Cardinal quantifiers

E.g. “more than 2”, “less than 7”, and “between 8 and 11”

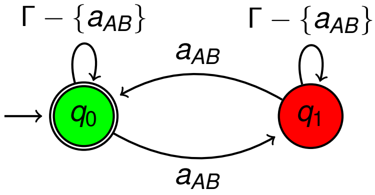


Finite automaton recognizing $L_{\text{More than two}}$

$$L_{\text{More than two}} = \{\alpha \in \Gamma^* : \#a_{AB}(\alpha) > 2\}$$

Parity quantifiers

E.g. “an even number”, “an odd number”

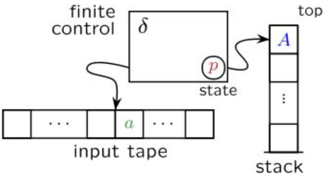


Finite automaton recognizing L_{Even}

$$L_{\text{Even}} = \{\alpha \in \Gamma^* : \#a_{AB}(\alpha) \text{ is even}\}$$

Proportional quantifiers

- E.g. "most", "less than half".
- Most *As are B* iff card$(A \cap B) >$ card$(A - B)$.
- $L_{\text{Most}} = \{\alpha \in \Gamma^* : \#a_{AB}(\alpha) > \#a_{A\bar{B}}(\alpha)\}$.
- There is no finite automaton recognizing this language.
- We need internal memory.
- A push-down automata will do.



Summing up

Definability	Examples	Recognized by
FO	“all” “at least 3”	acyclic FA
FO(D_n)	“an even number”	FA
PrA	“most”, “less than half”	PDA

Quantifiers, definability, and complexity of automata

Van Benthem, Essays in logical semantics, 1986.

Mostowski, Computational semantics for monadic quantifiers, 1998.

Does it say anything about processing?

Question

Do minimal automata predict differences in verification?

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We'll try to convince you that the answer is positive!

Specific task: ‘Most’ vs. ‘More than half’

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- Different distribution in corpus.
- They trigger different verification strategies.

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We saw data on ‘most’; let’s look into ‘more than half’.

Outline

Predictions

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- RT will increase along with the computational resources.
- Aristotelian qua. < parity qua. < proportional qua.
- Aristotelian qua. < cardinal qua. of high rank.

Materials

Grammatically simple propositions in Polish, like:

1. Some cars are red.
2. More than 7 cars are blue.
3. An even number of cars is yellow.
4. Less than half of the cars are black.

Materials continued

More than half of the cars are yellow.

An example of a stimulus used in the first study

Procedure

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  - “less than half” and “more than half” (PDA).

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- Quantity of target items near the criterion of validation.

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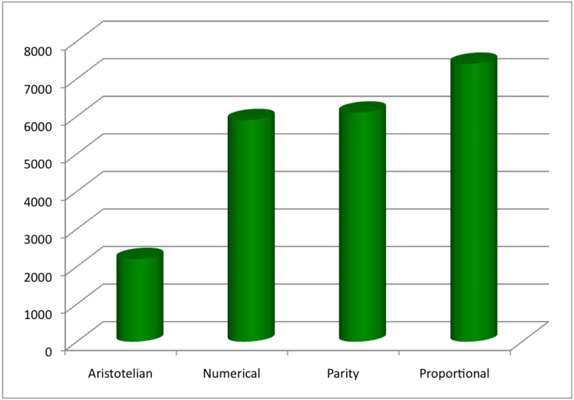
  - “odd” and “even” (2-state FA);
  - “less than 8” and “more than 7” (FA);
  - “less than half” and “more than half” (PDA).
- Quantity of target items near the criterion of validation.
- Numerical and proportional quantifiers logically equivalent.

Accuracy

Quantifier group	Examples	Percent
Aristotelian	all, some	99
Parity	odd, even	91
Cardinal	less than 8, more than 7	92
Proportional	less than half, more than half	85

The percentage of correct answers

RT



Szymaniki & Zajenkowski, Comprehension of simple quantifiers. Empirical evaluation of a computational model, Cognitive Science, 2010

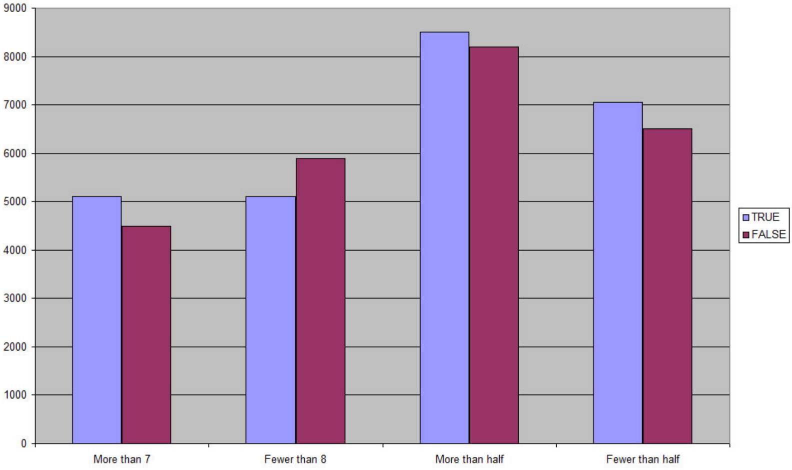
It interacts with monotonicity

1. “More than 7”: true > false (8>7).

It interacts with monotonicity

1. “More than 7”: true > false (8>7).
2. “Fewer than 8”: true < false (7<8).

Interaction effect



Szymanik & Zajenkowski,Computational approach to monotonicity in sentence-picture verification, under review

Outline

McMillan et al. fMRI studies

Differences in brain activity.

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- All quantifiers are associated with numerosity: recruit right inferior parietal cortex.

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Differences in brain activity.

- All quantifiers are associated with numerosity: recruit right inferior parietal cortex.
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But definability seems not to be fine grained enough!

McMillan et al., Neural basis for generalized quantifiers comprehension, Neuropsychologia, 2005

Szymanik, A Note on some neuroimaging study of natural language quantifiers comprehension, Neuropsychologia, 2007

Baddeley's model

WM unified system responsible for the performance in complex tasks.

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WM unified system responsible for the performance in complex tasks.

- The model consists of:
  - temporary storage units:
    - phonological loop;
    - visual loop;
  - a controlling system (central executive).

Baddeley, Working memory and language: an overview, 2003

Span test

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- To asses the working memory construct.

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  - the number of correctly memorized words?
  - the degree of understanding?
- Engagement of processing and storage functions.

Daneman and Carpenter, Individual differences in working memory, 1980

‘Computational’ theory of WM

Observation

A trade-off between processing and storage functions.

‘Computational’ theory of WM

Observation

A trade-off between processing and storage functions.

Hypothesis

One cognitive resource – competition for a limited capacity.

Daneman and Merikle, Working memory and language comprehension, 1996

Experimental setup

Question

How additional memory load influences quantifier verification?

Experimental setup

Question

How additional memory load influences quantifier verification?

Combined task:

- memorize sequences of digits;
- verify quantifier sentences;
- recall digits.

Results

- Trade-off effect only for PQs.
- WM engagement in PQs is qualitatively different.

Szymanik & Zajenkowski,Quantifiers and working memory, LNCS, 2010

Szymanik & Zajenkowski,Contribution of working memory in the parity and proportional judgments, submitted

Further evidence

- Compare performance of:

Further evidence

- Compare performance of:
  - Healthy subjects.

Further evidence

- Compare performance of:
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  - Patients with schizophrenia.

Further evidence

- Compare performance of:
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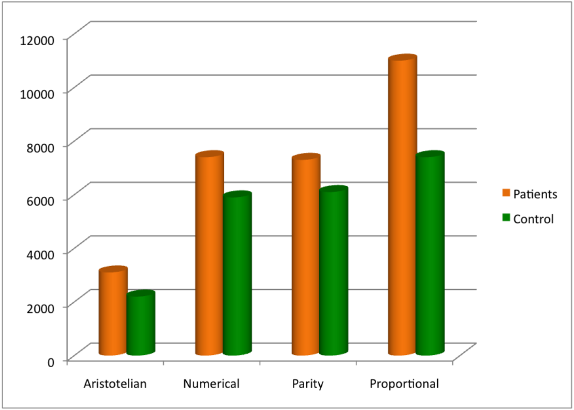
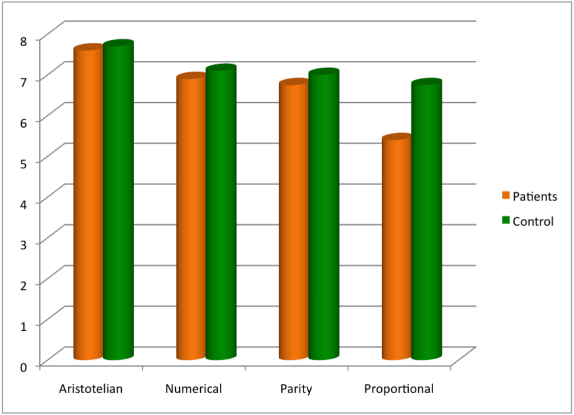
  - Patients with schizophrenia.
    - Known WM deficits.

RT data


12000
10000
8000
6000
4000
2000
0
Aristotelian
Numerical
Parity
Proportional
Patients
Control

Accuracy data



Zajenkowski et al., A computational approach to quantifiers as an explanation for some language impairments in schizophrenia, under review.

Summary

Conclusion

Automata model is psychologically plausible.

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Computational complexity $\approx$ cognitive difficulty.

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Conclusion

Precision with proportional quantifiers is cognitively challenging.

Outline

Bigger picture

- Enrich the model:
  1. Approximate Number System;
  2. Visual clues;

Pietroski et al., The Meaning of 'Most': semantics, numerosity, and psychology, Mind and Language, 1999

Lidz et al., Interface transparency and the psychosemantics of 'most', Natural Language Semantics, in press

Neurocognitive computational modeling

- Mechanism selection;
  - ‘more than half’ vs. ‘most’

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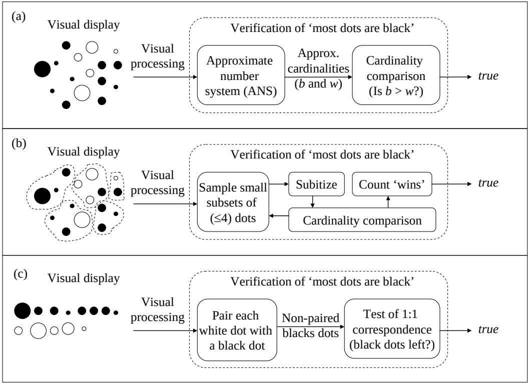
Neurocognitive computational modeling

- Mechanism selection;
  - ‘more than half’ vs. ‘most’
- Translate to neurocognitive setting, e.g.;
  - ACT-R modeling;
- Behavioral experiments:
  - determining factors influencing meaning selection.
- fMRI experiments.

Dehaene & Cohen, Cultural recycling of cortical maps, Neuron, 2007

Modeling example



Big THANKS to Organizers