Monotonicity in quantifier verification

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Outline

Quantifiers and Monotonicity

Monotonicity and Negativity Confound

Sentence-picture Verification Models
- Comparison model
- Automata Model

Experimental Hypotheses

Experiments

Results

Discussion

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NL determiners

1. All poets have low self-esteem.
2. Some dean danced nude on the table.
3. At least 3 grad students prepared presentations.
4. An even number of the students saw a ghost.
5. Most of the students think they are smart.
6. Less than half of the students received good marks.

Monotone quantifiers

Definition

Q is upward monotone if $X \subseteq Y$, then Q(X) entails Q(Y).

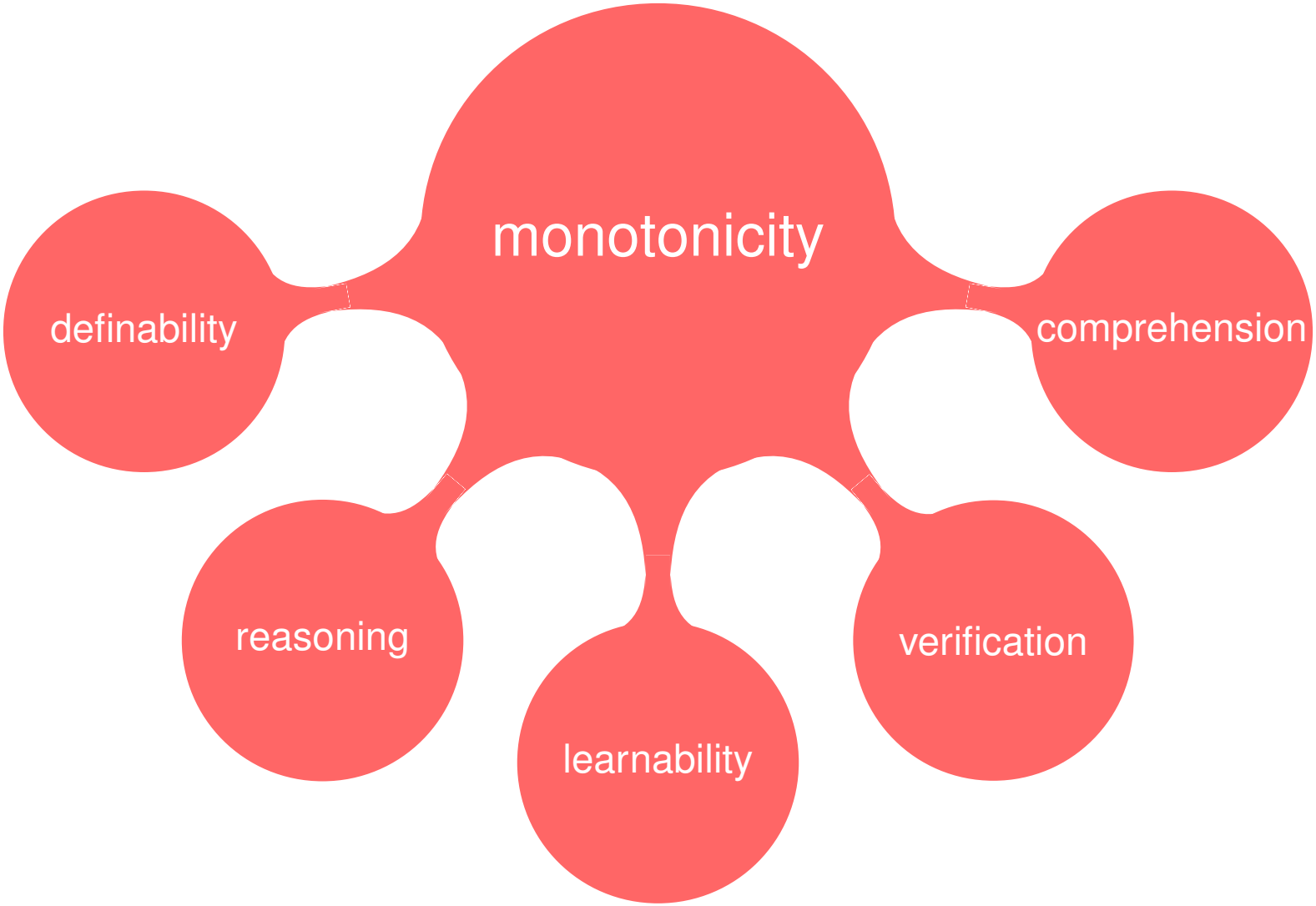
1. Every boy runs fast.
2. Every boy runs.

Definition

Q is downward monotone if $Y \subseteq X$, then Q(X) entails Q(Y).

1. No boy runs.
2. No boy runs fast.

Monotonicity – key property in logic and language



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Just and Carpenter 1971

Observation

Processing time of negative quantifiers is greater than processing time of affirmative quantifiers.

Just & Carpenter, Comprehension of negation with quantification, Journal of Verbal Learning and Verbal Behavior, 1971

3 kinds of sentences

1. Syntactic negatives with particle:
   - The dots are red.
   - The dots aren't red.
2. Syntactic negatives without particle:
   - Many of the dots are red.
   - Few of the dots are red.
3. Semantic negatives:
   - A majority of the dots are red.
   - A minority of the dots are red.

Only some pairs contrasted w.r.t. monotonicity:

1. All of the dots are red.
2. None of the dots are red.

Most of the material was based on negativity vs. affirmativity.

Affirmativity and monotonicity?

- Monotonicity is a semantic property of quantifiers;
- Degree of affirmativity is a linguistic concept, e.g.,:
  - tag test;
  - licensing NPIs.

Example

1. Few children are dirty, are they?
2. Few children believe that any more.
3. *A few children are dirty, are they?
4. *A few children believe that any more.

A partial dissociation

Observation

Dissociation between downward monotonicity and negativity.

Example

1. At most half of the children believe that, don't they?
2. Not many children believe that, do they?

Observation

Downward monotone quantifiers fall into two classes: affirmatives and negatives.

The confound

Question

Are effects reported by JC'71 due to monotonicity or negativity?

- It's common to mix up monotonicity and negativity.
- Our aim is to clearly separate those two:
  - Polish has no negativity degrees.
- Focus on monotonicity.

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Comparison model

4 stage processing of the comparison model:

1. Sentence encoding,
2. Picture encoding,
3. Comparing,
4. Responding.

Component latencies are additive.

Clark & Chase, On the process of comparing sentences against pictures, Cognitive Psychology, 1972

Pros & Cons of the comparison model

Pros

- *Explains variations w.r.t. negativity:*
  - *negatives harder than affirmatives;*
  - *true affirmatives easier than false ones;*
  - *false negatives easier than true ones.*

Cons

- *How to cover quantifiers?*
- *It says very little about monotonicity.*
- *Arbitrary psychological representation.*
- *Little insight into the actual computational process.*

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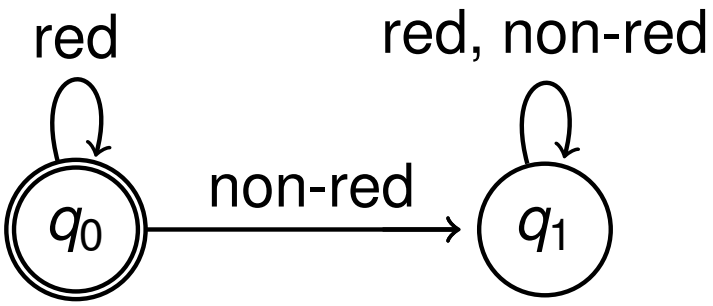
Simplicity

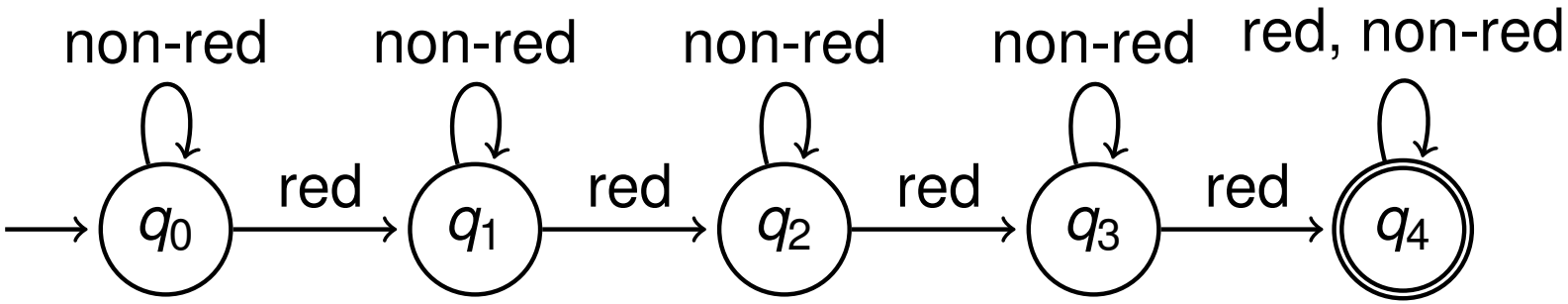
- Simple quantifiers can be computed by simple automata.
- Encoding natural counting strategies.
- We restrict ourselves to precise counting.

van Benthem, Essays in logical semantics, 1986

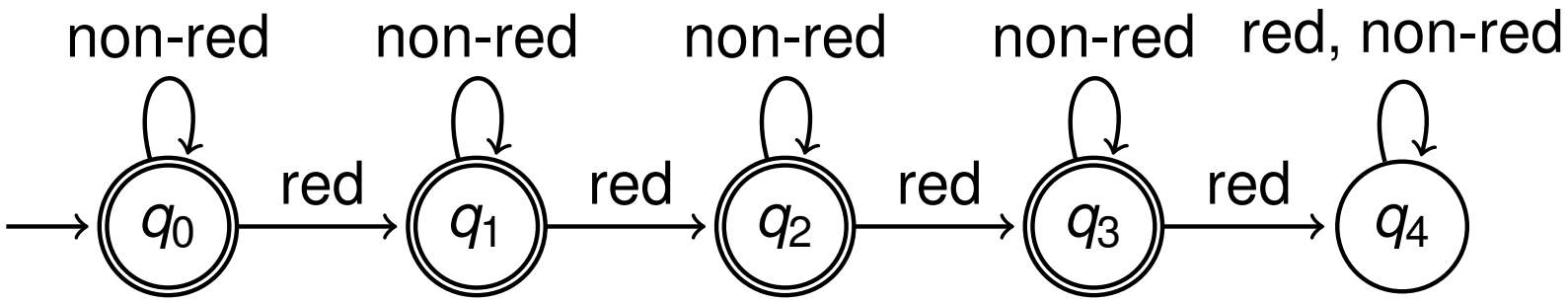
Every dot is red.



More than 3 dots are red.

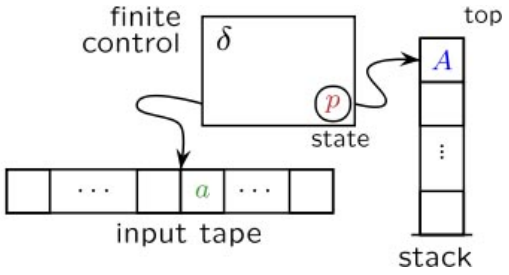


Fewer than 4 dots are red.



Proportional quantifiers

1. More than half of the dots are red.
2. Fewer than half of the dots are red.



- Not computable by finite-automata.
- We need working memory.
- Simple push-down automata will do.

Psychological plausibility

Observation

The more complex automata the longer reaction time and greater working memory involvement.

- McMillan et al., Neural basis for generalized quantifiers comprehension, Neuropsychologia, 2005
- Szymanik, A Note on some neuroimaging study of natural language quantifiers comprehension, Neuropsychologia, 2007
- Szymaniki & Zajenkowski, Comprehension of simple quantifiers. Empirical evaluation of a computational model, Cognitive Science, 2010
- Szymanik & Zajenkowski, Quantifiers and working memory, LNCS, 2010

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Design

- 4 different quantifiers:
  - Cardinal quantifiers (“more than 7”,“fewer than 8”);
  - Proportional (“more than half”, “fewer than half”).
- Upward monotone vs. downward monotone.
- True vs. false.
- Subjects were timed when asked to decide if true.

Predictions

1. RT increase along with the computational complexity.
2. Complexity influenced by (monotonicity $\times$ truth-value):
   - In the case of the cardinal sentences,
   - but not the proportional sentences.

Predictions in details

1. “More than 7”: true > false (8>7).
2. “Fewer than 8”: true < false (7<8).
3. On average “fewer than 8” > “more than 7”.
4. No difference between proportional quantifiers.
5. Proportional quantifiers > cardinal quantifiers.

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2 experiments

- 2 independent experiments.
- Same results.
- We report only on the first one.

Participants

- 30 native Polish-speaking adults (15 female).
- Volunteers: undergraduates from the University of Warsaw.
- The mean age: 21.30 years (SD = 2.54).
- Each participant tested individually.

Materials

16 grammatically simple propositions in Polish, like:

1. More than 7 cars are blue.
2. Fewer than 8 cars are yellow.
3. More than half of the cars are red.
4. Fewer than half of the cars are black.

Materials continued

More than half of the cars are yellow.

An example of a stimulus used in the first study

Procedure

- Each quantifier was presented in 10 trials.
- The sentence true in the picture in half of the trials.
- Quantity of target items near the criterion of validation.
- Practice session followed by the experimental session.
- Each quantifier problem was given one 15.5 s event.
- Subjects were asked to decide the truth-value.

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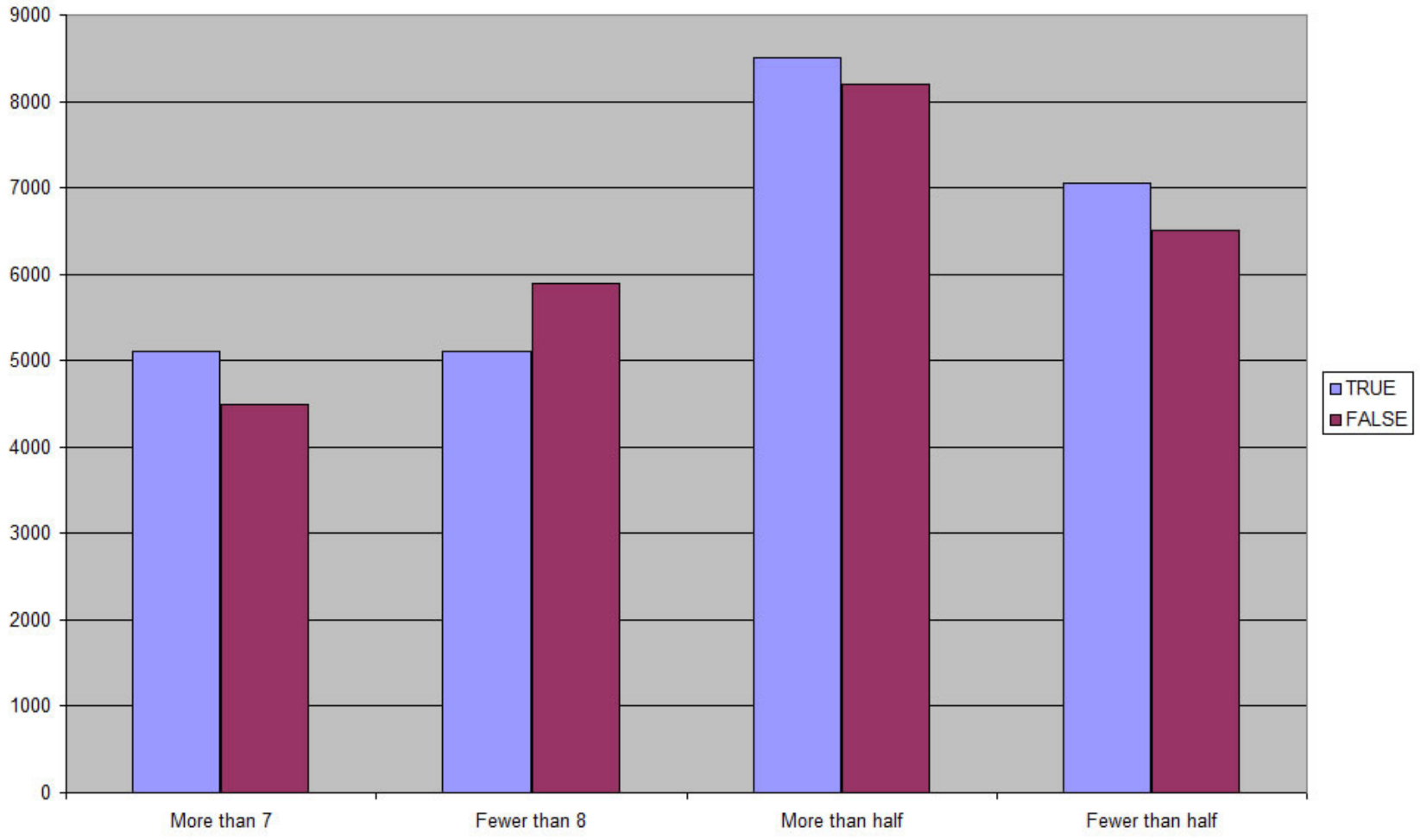
Raw data

Quantifier		M	SD
More than 7	True	5073.36	1244
	False	4526.36	1299
	Overall	4799.87	1157
Fewer than 8	True	5047.80	1693
	False	5872.13	1795
	Overall	5459.97	1510
More than half	True	7431.12	2076
	False	7195.13	2342
	Overall	7313.42	2016
Fewer than half	True	7064.80	2433
	False	6539.67	2092
	Overall	6802.23	2098

Interaction effect



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cf. Just and Carpenter, 1971

- Difference between cardinal quantifiers;
- No difference between proportional quantifiers.
- Differences in JC'71 is due to negativity.
- Vague quantifiers of JC'71 trigger different strategies.

False/True interaction

- $RT(\exists_T^{\geq 7}) > RT(\exists_F^{\geq 7})$? 👍
- $RT(\exists_T^{\leq 8}) < RT(\exists_F^{\leq 8})$. 👍
- No difference in proportional quantifiers. 👍
- Complexity hypothesis. 👍

Model's plausibility

- Complexity hypothesis derived from the minimal automata.
- Proportional quantifiers > numerical quantifiers.
- Monotonicity accounts for 45% of the variance.
- Complexity for over 90%.
- Complexity explains more than monotonicity.

Thank you for attention