Muddy Children Playground

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LoRI @ ESSLLI'10

The puzzle

1. At least one of you has mud on your forehead.
2. Can you tell whether or not you are muddy?

Repeating the question makes children know the answer.

The First Announcement: Quantifier

General form:

'Q of you have mud on your forehead',

where Q is a generalized quantifier.

Quantifiers of type (1)

$$M = (U, A)$$

After the announcement:

$$\{M : M \models \mathsf{Q}_U(A)\}$$

Examples

Example

$$\exists = \{(U, A) : A \subseteq U \;\&\; A \neq \emptyset\}$$

$$\mathrm{D_n} = \{(U, A) : A \subseteq U \;\&\; card(A) = k \times n\}$$

$$\mathrm{most} = \{(U, A) : A \subseteq U \;\&\; card(A) > card(U - A)\}$$

Number triangle

Viewing finite models as pairs of integers.

				(0, 0)				
			(1, 0)		(0, 1)			
		(2, 0)		(1, 1)		(0, 2)		
	(3, 0)		(2, 1)		(1, 2)		(0, 3)	
(4, 0)		(3, 1)		(2, 2)		(1, 3)		(0, 4)

....

				—				
			-		+			
		-		+		+		
	-		+		+		+	
-		+		+		+		+

....

Extensively studied in GQT.

General form:

‘Q of you have mud on your forehead’,

where Q is a generalized quantifier.

Varying Quantifiers in Muddy Children Puzzle

'At least one' and 'At least two'

	0	1	2	3	4
1	x	1	x	x	x
2	x	1	2	x	x
3	x	1	2	3	x
4	x	1	2	3	4
5	x	1	2	3	4
6	x	1	2	3	4 ...

	0	1	2	3	4
1	x	x	x	x	x
2	x	x	1	x	x
3	x	x	1	2	x
4	x	x	1	2	3
5	x	x	1	2	3
6	x	x	1	2	3 ...

‘Even’ and ‘Most’

	0	1	2	3	4
1	1	x	1	x	1
2	1	x	1	x	1
3	1	x	1	x	1
4	1	x	1	x	1
5	1	x	1	x	1
6	1	x	1	x	1 . . .

	0	1	2	3	4	5
1	x	1	x	x	x	x
2	x	x	1	x	x	x
3	x	x	1	2	x	x
4	x	x	x	1	2	x
5	x	x	x	1	2	3
6	x	x	x	x	1	2 . . .

Patterns: Exemplary observation

Proposition

Assume n children, $m \leq n$ muddy children. The Muddy Children Puzzle with the background assumption 'At least k of you have mud on your forehead' can be solved in $m - (k - 1)$ steps, where $k \leq m$.

In the paper we do it systematically for various Qs.

WHERE DO THOSE PATTERNS COME FROM?

'At least one'

	0	1	2	3	4	5
1	x	1	x	x	x	x
2	x	1	2	x	x	x
3	x	1	2	3	x	x
4	x	1	2	3	4	x
5	x	1	2	3	4	5
6	x	1	2	3	4	5 ...

THEY COME FROM THE QUANTIFIER ITSELF

—

\- 1

\- 1 2

\- 1 2 3

\- 1 2 3 4

\- 1 2 3 4 5

. . . .

THEOREM

Let n be the number of children, $m \leq n$ the number of muddy children, and Q be the background assumption. Muddy Children situation is solvable iff $(n-m, m) \in Q$ and there is an $l \leq n$ such that $(n-l, l) \notin Q$.

$$\begin{array}{ccccccccccc}
 & & & & & - & & & & & \\
 & & & ? & & & & ? & & & \\
 & & ? & & & ? & & & ? & & \\
 & 3 & & & 2 & & & 1 & & - & \\
3 & & & 2 & & & 1 & & - & & -
\end{array}$$

‘At most 2’

Solvability characterization

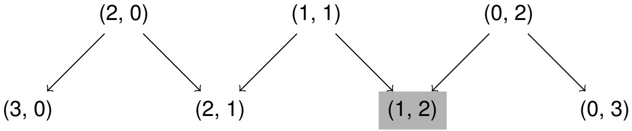
Observation

The number assigned to a point in the number triangle is the 'distance' to the closest model outside of the quantifier.

```
            —
         ?     ?
      ?     ?     ?
   3     2     1     -
3     2     1     -     -
```

'At most 2'

Representation



Observation

Every agent's observation is encoded by one of at most two neighboring states in the observational level.

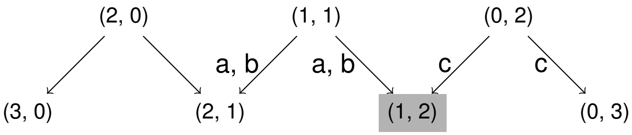
Representation

$$
\begin{tikzcd}
 & (2,0) \arrow[dl] \arrow[dr] & & (1,1) \arrow[dl] \arrow[dr] & & (0,2) \arrow[dl] \arrow[dr] & \\
(3,0) & & (2,1) & & \boxed{(1,2)} & & (0,3)
\end{tikzcd}
$$

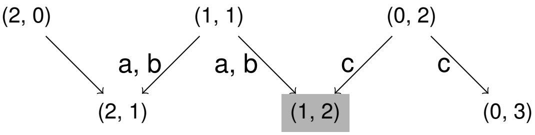
Observation

Every agent's observation is encoded by one of at most two neighboring states in the observational level.

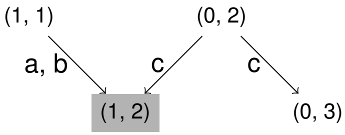
Initial Model



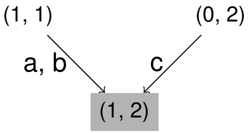
Step 1: Quantifier Announcement



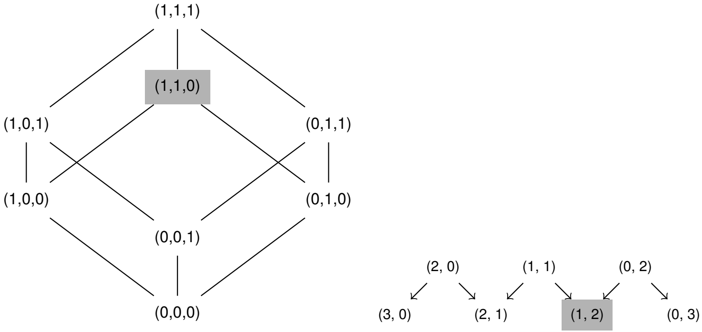
Step 2: Epistemic Reasoning



Step 3: Epistemic Reasoning



Muddy Children Modeling DEL vs NT (CogSci)



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Discussion/Outlook

- Comparison with DEL-perspective.
- Isomorphism and symmetry.
- Associate our representations with automata.
- Logic for public announcements with GQs.
- Other epistemic puzzles.