A REMARK ON COLLECTIVE QUANTIFICATION

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The 9th Szklarska Poręba Workshop
February 22, 2008
The common strategy in formalizing collective quantification has been to define the meanings of collective determiners using certain type-shifting operations. These type-shifting operations, i.e., lifts, define the collective interpretations of determiners systematically from the standard meanings of quantifiers. We argue that this approach is probably not expressive enough to formalize all collective quantification in natural language!
INTRODUCTION

LIFTING FIRST-ORDER DETERMINERS

GENERALIZED QUANTIFIERS

DEFINING COLLECTIVE DETERMINERS

LIFTING THE DETERMINER MOST

CONCLUSION
Introduction
Lifting first-order determiners
Generalized quantifiers
Defining collective determiners
Lifting the determiner most
Conclusion
Motivations

• Complexity depends on the quantifiers.
• Mainly distributive determiners are considered.
• However, plural objects are becoming important.
• E.g. in game-theory, where groups of agents are acting.
(1.) Tikitu and Samson lifted the poker table together.
(2.) Most groups of students played Hold’em together.
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Let’s start with examples

(1.) Five people lifted the table.
(1’.) $\exists=^5 x [\text{People}(x) \land \text{Lift}(x)]$.
(1’’.)$\exists X [X \subseteq \text{People} \land \text{Card}(X) = 5 \land \text{Lift}(X)]$.

(2.) Some students played poker together.
(2’.)$\exists X [X \subseteq \text{Students} \land \text{Play}(X)]$. 
**EXISTENTIAL MODIFIER**

**DEFINITION**

Fix a universe of discourse $U$ and take any $X \subseteq U$ and $Y \subseteq \mathcal{P}(U)$. Define the existential lift $Q^{EM}$ of a quantifier $Q$ in the following way:

$$ Q^{EM}(X, Y) \text{ is true } \iff \exists Z \subseteq X[Q(X, Z) \land Z \in Y]. $$

$$((et)((et)t)) \rightsquigarrow ((et)(((et)t)t))$$
Determiner Fitting

Definition

For all $X, Y \subseteq \mathcal{P}(U)$ we have that

$$Q_{dfit}(X, Y) \text{ is true}$$

$$\iff$$

$$Q[\cup X, \cup (X \cap Y)] \land [X \cap Y = \emptyset \lor \exists W \in X \cap Y \land Q(\cup X, W)].$$

$$(et)((et)t) \leadsto (((et)t)(((et)t)t))$$
OUTLINE

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LINDSTRÖM QUANTIFIERS

\[ \forall = \{(M, P) | P = M\} \]
\[ \exists = \{(M, P) | P \subseteq M \& P \neq \emptyset\} \]
\[ \text{even} = \{(M, P) | P \subseteq M \& \text{card}(P) \text{ is even}\} \]
\[ \text{most} = \{(M, P, S) | P, S \subseteq M \& \text{card}(P \cap S) > \text{card}(P - S)\} \]
\[ \text{some} = \{(M, P, S) | P, S \subseteq M \& P \cap S \neq \emptyset\} \]
SECOND-ORDER GQS

\[ \exists^2 = \{(M, P) \mid P \subseteq \mathcal{P}(M) \& P \neq \emptyset \} \]

\[ \text{EVEN} = \{(M, P) \mid P \subseteq \mathcal{P}(M) \& \text{card}(P) \text{ is even} \} \]

\[ \text{EVEN}' = \{(M, P) \mid P \subseteq \mathcal{P}(M) \& \forall X \in P(\text{card}(X) \text{ is even}) \} \]

\[ \text{MOST} = \{(M, P, S) \mid P, S \subseteq \mathcal{P}(M) \& \text{card}(P \cap S) > \text{card}(P \setminus S) \} \]
WARNING

Do not confuse:

- FO GQs (Lindström) with FO-definable quantifiers
  
  E.g. most is FO GQs but is not FO-definable.

- SO GQs with SO-definable quantifiers

  E.g. Most is SO GQs but probably not SO-definable.
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**For example...**

**Definition**

Denote by some\(^{EM}\): 

\[ \{(M, P, G) \mid P \subseteq M; \ G \subseteq \mathcal{P}(M) : \exists Y \subseteq P(Y \neq \emptyset & P \in G)\}. \]

(3.) Some students played poker together.

(3’.) some\(^{EM}\) \(x, X[\text{Student}(x), \text{Play}(X)].\)
**Definition**

We take $\text{five}^{EM}$ to be the second-order quantifier denoting:

$$\{(M, P, G) \mid P \subseteq M; \ G \subseteq \mathcal{P}(M) : \exists Y \subseteq P(\text{card}(Y) = 5 \& P \in G)\}.$$

(4.) Five people lifted the table.
(4’.) $\text{five}^{EM}x, X[\text{Student}(x), \text{Lift}(X)]$. 
GENERAL OBSERVATION

THEOREM

Let $Q$ be a first-order quantifier definable in $SO$. Then the second-order quantifiers $Q^{EM}$ and $Q^{dfit}$ are definable in $SO$.

PROOF.

Let us consider the case of $Q^{EM}$. Let $\psi(x)$ and $\phi(Y)$ be formulas. We want to express $Q^{EM}x, Y(\psi(x), \phi(Y))$ in second-order logic. By the assumption, the quantifier $Q$ can be defined by some sentence $\theta \in SO[\{P_1, P_2\}]$. We can now use the following formula:

$$\exists Z(\forall x(Z(x) \rightarrow \psi(x)) \land (\theta(P_1/\psi(x), P_2/Z) \land \phi(Y/Z)).$$
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(5.) Most groups of students played Hold’em together.

(5’.) $\textsc{Most } X, Y[\text{Students}(X), \text{Play}(Y)]$.

- The discussed lifts do not give the intended meaning.
- It is unlikely that any lift can do the job.
- Otherwise, highly unexpected things would happen!
THE MAIN THEOREM

THEOREM

If the quantifier MOST is definable in second-order logic, then counting hierarchy, CH is equal polynomial hierarchy, PH. Moreover, CH collapses to its second level.

PROOF.

The logic FO(MOST) can define complete problems for each level of the CH (Kontinen&Niemisto’06). If MOST would be definable in SO, then FO(MOST) ≤ SO and therefore SO would contain complete problems for each level of the CH. This would imply that CH = PH and furthermore that CH ⊆ PH ⊆ C2P. □
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TO SUM UP

- We can approach collectivity in terms of SO GQs.
- The previous attempts have relied on SO-definable GQs.
- This approach is probably not general enough.
- We observed this by studying computational complexity.
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