Comprehension of simple quantifiers Empirical evaluation of a computational model

Jakub Szymanik

Institute for Logic, Language and Computation
Universiteit van Amsterdam

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ABSTRACT

- Comprehension of simple quantifiers in natural language.
- Computational model posited by many logicians.
- Linking computational complexity and cognitive science.
- Comparing RT needed for understanding:
  - FA-quantifiers vs. PDA-quantifiers;
  - Aristotelian quantifiers vs. cardinal quantifiers;
  - Parity quantifiers;
  - PDA-quantifiers over ordered and unordered universes.

OUTLINE

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Computability and cognition

- A cognitive task is a computational task.

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Frixione, "Tractable competence", 2001

van Rooij, "The tractable cognition thesis", 2008

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- But not enough empirical links, too abstract considerations.

MEANING AS ALGORITHM

- Ability of understanding sentences.
- Capacity of recognizing their truth-values.

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Meaning as algorithm

- Ability of understanding sentences.
- Capacity of recognizing their truth-values.
- Long-standing philosophical (Fregean) tradition.
- Meaning is a procedure for finding extension in a model.
- Adopted often with psychological motivations.

Suppes, "Variable-free semantics with remark on procedural extensions", 1982

Lambalgen & Hamm, "The proper treatment of events", 2005

Previous investigations

Brain activity during the comprehension of:

FO-quantifiers vs. higher-order quantifiers.

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FO-quantifiers vs. higher-order quantifiers.

Results:

- All quantifies are associated with numerosity: recruit right inferior parietal cortex;
- Only higher-order activate working-memory capacity: recruit right dorsolateral prefrontal cortex;

McMillan et al., "Neural basis for generalized quantifiers comprehension", 2005

Clark & Grossman, "Number sense and quantifier interpretation", 2007

Additional support

- Corticobasal degeneration (CBD) — number knowledge.
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- Corticobasal degeneration (CBD) — number knowledge.
- Alzheimer (AD) and frontotemporal dementia (FTD) — working memory limitations.
- CBD impairs comprehension more than AD and FTD.
- FTD and AD patients have greater difficulty in non-FO.

McMillan et al., "Quantifiers comprehension in corticobasal degeneration", 2006

Problems

- Definability $\neq$ Complexity
- Computational differences missed;
- "Even" is higher-order but FA-computable.
- Complexity perspective is better grained.
- New experimental set up!

Szymanik, "A note on a neuroimaging study of natural language quantifiers comprehension", 2007

Szymanik and Zajenkowski, "Improving methodology of quantifier comprehension experiments", 2009

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   - Generalized Quantifiers
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Simple quantifier sentences

- Every poet has low self-esteem.
- Some dean danced nude on the table.
- At least 3 grad students prepared presentations.
- An even number of the students saw a ghost.
- Most of the students think they are smart.
- Less than half of the students received good marks.

Lindström definition

Definition

A monadic generalized quantifier of type (1,1) is a class Q of structures of the form $M = (U, A_1, A_2)$, where $A_1, A_2 \subseteq U$. Additionally, Q is closed under isomorphism.

A few examples

- some $= \{(U, A, B) : A, B \subseteq U \wedge A \cap B \neq \emptyset\}$

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- some $= \{(U, A, B) : A, B \subseteq U \wedge A \cap B \neq \emptyset\}$
- all $= \{(U, A, B) : A, B \subseteq U \wedge A \subseteq B\}$
- exactly m $= \{(U, A, B) : A, B \subseteq U \wedge \text{card}(A \cap B) = m\}$

A FEW EXAMPLES

- some $= \{(U, A, B) : A, B \subseteq U \wedge A \cap B \neq \emptyset\}$
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- exactly m $= \{(U, A, B) : A, B \subseteq U \wedge \text{card}(A \cap B) = m\}$
- even $= \{(U, A, B) : A, B \subseteq U \wedge \text{card}(A \cap B) = k \times 2\}$
- most $= \{(U, A, B) : \text{card}(A \cap B) > \text{card}(A - B)\}$

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HOW DO WE ENCODE MODELS?

- Restriction to finite models of the form $M = (U, A, B)$.

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- α_M describes the model in which: $c_1 \in \bar{A}\bar{B}, c_2 \in A\bar{B} c_3 \in AB, c_4 \in \bar{A}B, c_5 \in \bar{A}B$.

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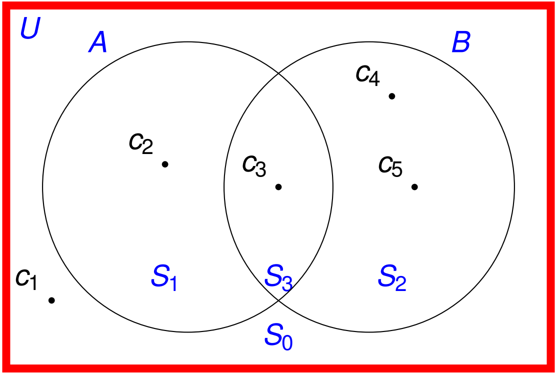
- The class Q is represented by the set of words describing all elements of the class.

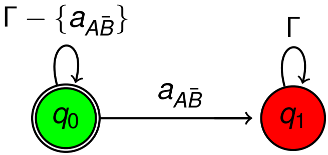
ILLUSTRATION



This model is uniquely described by $\alpha_M = a_{\bar{A}\bar{B}} a_{A\bar{B}} a_{AB} a_{\bar{A}B} a_{\bar{A}B}$

Aristotelian quantifiers

“all”, “some”, “no”, and “not all”

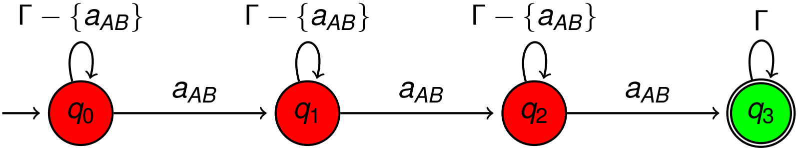


Finite automaton recognizing L_{All}

$$L_{\text{All}} = \{\alpha \in \Gamma^* : \# a_{A\bar{B}}(\alpha) = 0\}$$

Cardinal quantifiers

E.g. “at least 3”, “at most 7”, and “between 8 and 11”

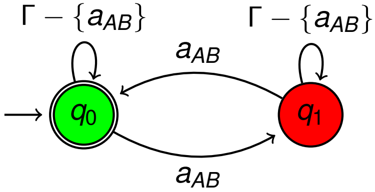


Finite automaton recognizing $L_{\text{At least three}}$

$$L_{\text{At least three}} = \{\alpha \in \Gamma^* : \#a_{AB}(\alpha) \geq 3\}$$

Parity quantifiers

E.g. “an even number”, “an odd number”



Finite automaton recognizing L_{Even}

$$L_{\text{Even}} = \{\alpha \in \Gamma^* : \#a_{AB}(\alpha) \text{ is even}\}$$

PROPORTIONAL QUANTIFIERS

- E.g. "most", "less than half".
- Most *As are B* iff $\mathrm{card}(A \cap B) > \mathrm{card}(A - B)$.
- $L_{\mathrm{Most}} = \{\alpha \in \Gamma^* : \#a_{AB}(\alpha) > \#a_{A\bar{B}}(\alpha)\}$.
- There is no finite automaton recognizing this language.
- We need internal memory.
- A push-down automata will do.

What does it mean that class of monadic quantifiers is recognized by class of devices?

Definition

Let $\mathcal{D}$ be a class of recognizing devices,
Ω a class of monadic quantifiers.
We say that $\mathcal{D}$ accepts Ω if and only if
for every monadic quantifier Q:

$$\mathrm{Q} \in \Omega \iff \text{there is device } A \in \mathcal{D}(A \textit{ accepts } L_{\mathrm{Q}}).$$

In General

Definability	Examples	Recognized by
FO	“all” “at least 3”	acyclic FA
FO(D_n)	“an even number”	FA
PrA	“most”, “less than half”	PDA

Quantifiers, definability, and complexity of automata

van Benthem, Essays in logical semantics, 1986

Mostowski, Computational semantics for monadic quantifiers, 1998

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Generalities

- Joint work with Marcin Zajenkowski.
- 1st: RT in the comprehension of different quantifiers.
- 2nd: engagement of working-memory capacity.

Szymanik and Zajenkowski, “Understanding quantifiers in language”, 2009

Szymanik and Zajenkowski, “Comprehension of simple quantifiers. Empirical evaluation of a computational model”, 2009

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General idea

- Compare RT wrt the following classes of quantifiers:

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  - recognized by acyclic FA (first-order);

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General idea

- Compare RT wrt the following classes of quantifiers:
  - recognized by acyclic FA (first-order);
  - not first-order recognized by FA (parity);
  - recognized by PDA but not FA.
- Additionally:
  - Aristotelian vs. cardinal quantifiers of higher rank.

Troiani et al., “Is it logical to count on quantifiers? Dissociable neural networks underlying numerical and logical quantifiers”, 2009

PREDICTIONS

- RT will increase along with the computational resources.
- Aristotelian qua. < parity qua. < proportional qua.
- Aristotelian qua. < cardinal qua. of high rank.
- Parity qua. < cardinal qua. of high rank.

PARTICIPANTS

- 40 native Polish-speaking adults (21 female).
- Volunteers: undergraduates from the University of Warsaw.
- The mean age: 21.42 years (SD = 3.22).
- Each participant tested individually.

MATERIALS

80 grammatically simple propositions in Polish, like:

1. Some cars are red.
2. More than 7 cars blue.
3. An even number of cars is yellow.
4. Less than half of the cars are black.

Materials continued

More than half of the cars are yellow.

An example of a stimulus used in the first study

PROCEDURE

- 8 different quantifiers divided into four groups.

Procedure

- 8 different quantifiers divided into four groups.
  - “all” and “some”;

Procedure

- 8 different quantifiers divided into four groups.
  - “all” and “some”;
  - “odd” and “even”;

Procedure

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  - “less than 8” and “more than 7”;

PROCEDURE

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  - “less than 8” and “more than 7”;
  - “less than half” and “more than half”.

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- Each quantifier was presented in 10 trials.

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- The sentence true in the picture in half of the trials.

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- Each quantifier problem was given one 15.5 s event.

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- The sentence true in the picture in half of the trials.
- Quantity of target items near the criterion of validation.
- Practice session followed by the experimental session.
- Each quantifier problem was given one 15.5 s event.
- Subjects were asked to decide the truth-value.

Analysis of accuracy

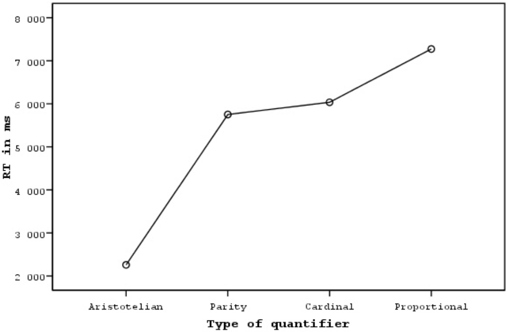
Quantifier group	Examples	Percent
Aristotelian FO	all, some	99
Parity	odd, even	91
Cardinal FO	less than 8, more than 7	92
Proportional	less than half, more than half	85

The percentage of correct answers

To sum up

- Increase in RT was determined by the quantifier type ($F(2.4, 94.3) = 341.24; p < 0,001; \eta^2 = 0.90$)
- Pairwise comparisons: all four types of quantifiers differed significantly from one another.
- The mean reaction time increased as follows: Aristotelian, parity, cardinal, proportional.

Comparison of reaction times



Average reaction times in each type of quantifiers

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General idea

- Investigating the role of working-memory capacity.
- The ordering as an additional independent variable.
- For example, consider the following sentence:
  "Most As are B."
- Universe ordered in pairs (a, b) such that $a \in A$, $b \in B$.

PREDICTIONS

- Given "good" ordering WM capacity is not needed.
- Ordering simplifies the problem = decrease in RT.

PARTICIPANTS

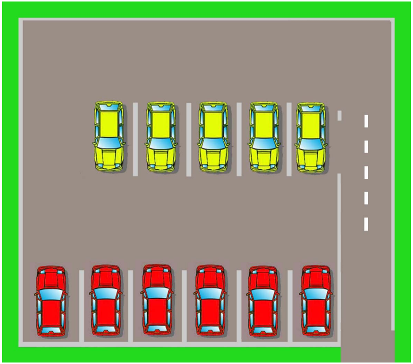
- 30 native Polish-speaking adults (18 females).
- Undergraduates from two Warsaw universities.
- The mean age: 23.4 years (SD = 2.51).
- Each subject tested individually.

Materials and procedure

- 16 grammatically simple propositions in Polish.
- E.g. “More than half of the cars are blue”.
- A car park with 11 cars.
- 2 quantifiers: “less than half” and “more than half”.
- Presented to each subject in 8 trials.
- Each type of sentence true in half of the trials.
- 4 ordered and 4 unordered pictures.
- The rest of the procedure the same as before.

Example of an ordered task

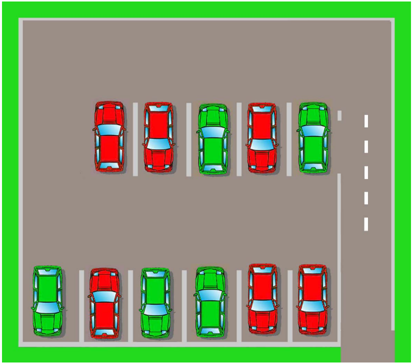
More than half of the cars are red.



A case when cars are ordered

Example of an unordered task

More than half of the cars are green.



A case when cars are distributed randomly

Results

- Higher accuracy of judgments for ordered universes (89%);
- Than for unordered (79%).
- Proportional quantifiers over randomized universes (M=6185.93; SD=1759.09);
- Over ordered models (M=4239.00; SD=1578.26);
- Hypothesis confirmed! ($t(29) = 5.87; p < 0,001; d = 1.16$).

Outline

Conclusions

- Plausibility of the model.
- Aristotelian easier than parity:
  loops influence the complexity of cognitive tasks.
- Cardinal harder than parity:
  number of states influences hardness more than loops.
- Proportional quantifiers involve working-memory capacity.
- Humans are constrained by computational resources.

PERSPECTIVES

- Comprehension and brain?

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- Comprehension strategies?

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- Comprehension and brain?
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- Comprehension and working memory?
- Comprehension and monotonicity?
- Comprehension beyond quantifiers?

Thank you!