Computational Complexity of the Reciprocal Lifts and Strong Meaning Hypothesis

Computational dichotomy between reciprocals

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Tbilisi Symposium
October 4, 2007

Abstract

- Study reciprocals, like “each other”.
- Define them as lifts over monadic GQs.
- Show computational dichotomy:
  - Strong r.l. over proportional quantifiers are NP-complete.
  - PTIME quantifiers are closed on intermediate and weak r.l.
- R.l. are frequent NP-complete constructions.
- Trying to justify SMH from those results.

Outline

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- Link semantics and computational complexity.
- Evaluate complexity of semantic constructions in order to:
  - better understand our linguistic competence.
  - investigate into robustness of linguistic distinctions.
- Classify semantic constructions by their complexity.
- It will be valuable for cognitive science.
- Clarify concept of “meaning”.

Outline

GQs — a short reminder

Definition

A generalized quantifier Q of type $(n_1, \ldots, n_k)$ is a class of structures of the form $M = (U, R_1, \ldots, R_k)$, where R_i is a subset of U^{n_i}. Additionally, Q is closed under isomorphism.

$$(U, R_1{}^M, \ldots, R_k{}^M) \in \mathrm{Q} \iff \mathrm{Q}_M R_1 \ldots R_k, \text{where } R_i{}^M \subseteq U^{n_i}.$$

Example

$$\mathrm{MOST} = \{(U, A^M, B^M) : card(A^M \cap B^M) > card(A^M - B^M)\}.$$

$$M \models \mathrm{MOST}_M AB \text{ iff } card(A^M \cap B^M) > card(A^M - B^M).$$

Quantifiers and complexity

Definition

Let Q be of type $(n_1, \ldots, n_k)$. By complexity of Q we mean computational complexity of the corresponding class K_{Q}.

Our computational problem is to decide whether $M \in K_{\mathrm{Q}}$.
Equivalently, does $M \models \mathrm{Q}[R_1, \ldots R_k]$?

Definition

We say that Q is NP-hard if K_{Q} is NP-hard.
Q is mighty if K_{Q} is NP and K_{Q} is NP-hard.

It was Blass and Gurevich 1986 who first studied those notions.

Previous results

Under branching interpretation the following sentences are NP-complete:

(1.) *Some relative of each villager and some relative of each townsman hate each other.*

(2.) *Most villagers and most townsmen hate each other.*

However, all these sentences are ambiguous and can be hardly found in the corpus of language.

Outline

Reciprocal expressions are common in English

(1.) *Andi, Jarmo and Jakub laughed at one another.*

(2.) *15 men are hitting one another.*

(3.) *Even number of the PMs refer to each other.*

(4.) *Most Boston pitchers sat alongside each other.*

(5.) *Some pirates were staring at each other in surprise.*

Various interpretations

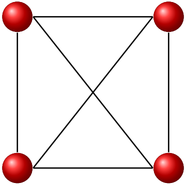
Dalrymple et al. 1998 classifies possible readings.
They explain variations in the meaning by:

Strong Meaning Hypothesis

Reading associated with the reciprocal in a given sentence is the strongest available reading which is consistent with relevant information supplied by the context.

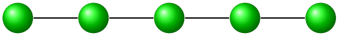
Strong reading

(3.) *Even number of the PMs refer to each other.*



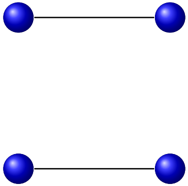
Intermediate reading

(4.) *Most Boston pitchers sat alongside each other.*



Weak reading

(5.) *Some pirates were staring at each other in surprise.*



And other possible variations...

(6.) Stones are arranged on top of each other.

So-called intermediate alternative reciprocity.

Outline

Strong reciprocal lift

Let Q be a monadic monotone increasing quantifier.

Definition

$\mathrm{Ram}_S(\mathrm{Q})AR \iff \exists X \subseteq A[\mathrm{Q}(X) \wedge \forall x, y \in X(x \neq y \Rightarrow R(x, y))]$.

Example

(3.) *Even number of the PMs refer to each other indirectly.*

(3'.) $\mathrm{Ram}_S(\mathrm{EVEN})$MP Refer.

Intermediate reciprocal lift

Definition

$$\mathrm{Ram_I}(\mathrm{Q})AR \iff \exists X \subseteq A[\mathrm{Q}(X) \wedge \forall x, y \in X$$
$$(x \neq y \Rightarrow \exists \text{ sequence } z_1, \ldots, z_\ell \in X \text{ such that}$$
$$(z_1 = x \wedge R(z_1, z_2) \wedge \ldots \wedge R(z_{\ell-1}, z_\ell) \wedge z_\ell = y)].$$

Example

(4.) *Most Boston pitchers sat alongside each other.*

(4'.) $\mathrm{Ram_I}(\mathrm{MOST})$Pitcher Sit.

Weak reciprocal lift

Definition

$$\mathrm{Ram}_{\mathrm{W}}(\mathrm{Q})AR \iff \exists X \subseteq A[\mathrm{Q}(X) \wedge \forall x \in X \exists y \in X (x \neq y \wedge R(x, y))].$$

Example

(5.) *Some pirates were staring at each other in surprise.*

(5'.) $\mathrm{Ram}_{\mathrm{W}}(\mathrm{SOME})$Pirate Staring.

Outline

Strong r.l. over counting quantifiers

(6.) At least 2 PMs refer to each other.

(7.) At least 7 PMs refer to each other.

(8.) At least k PMs refer to each other.

We just use: $\mathrm{Ram}_S(\exists^{\geq k})$

Strong r.l. over counting quantifiers is NP-complete

Definition

$M \models \exists^{\geq k} y \varphi(y)[v] \iff card(\varphi^{(M,y,v)}) \geq v(k).$

Proposition

Quantifier $\mathrm{Ram_S}(\exists^{\geq k})$ *is mighty.*

Proof.

$M \models \mathrm{Ram_S}(\exists^{\geq k}) AR$ if there is clique C s.t. $card(C) \geq v(k)$. $\square$

Strong r.l. over proportional quantifiers

(9.) Most PMs refer to each other.

(10.) At least one third of the PMs refer to each other.

(11.) At least $q \times 100\%$ of the PMs refer to each other.

Definition

$M \models \mathrm{R_q}xy\ \varphi(x,y)$ iff there is $A \subseteq U$ s. t. for all $a, b \in A$ $M \models \varphi(a,b)$ and A is q-big, i.e. $\frac{card(A)}{card(U)} \geq q$.

Proposition

Let $q \in]0,1[\cap \mathbb{Q}$, then the quantifier $\mathrm{R_q}$ is mighty.

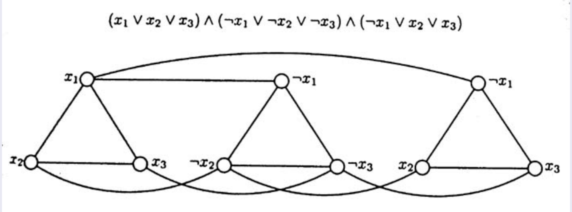
Proof of the Proposition 2

Corollary

q-big clique is NP-complete for $q \geq \frac{1}{k}$*, where* $k > 2$.

Proof.

It follows from the NP-completeness proof of INDEPENDENT SET. Consider graphs divided on complete disjoint k-agons.



Continuation of the proof

Lemma

For every $q \in]0,1[\cap \mathbb{Q}$ problem q-big clique is NP-complete.

Proof.

Let $G = (V, E)$ be s.t. $card(V) = ka$. In G exists $\frac{1}{k}$-big clique iff in G' exists $\frac{m}{k}$-big clique for $m < k$, where $G' = (V', E')$ is constructed as follows:

- $V' = V \cup U$, where U s.t. $card(U) = n = \lceil \frac{(m-1)ka}{k-m} \rceil$ and $U \cap V = \emptyset$;
- $E' = E \cup U \times (U \cup V)$.

It suffices to observe that $\frac{n+a}{n+ka} \geq \frac{m}{k} > \frac{n+(a-1)}{n+ka}$. □

Intermediate lift does not increase complexity

Proposition

If Q *is PTIME quantifier, then also* $\mathrm{Ram}_{\mathrm{I}}(\mathrm{Q})$ *is in PTIME.*

Proof.

To check whether $M \in \mathrm{Ram}_{\mathrm{I}}(\mathrm{Q})$ use breadth-first search algorithm to compute all connected components of M. Their number is bounded by $card(U)$.Then check whether $\mathrm{Q}(C)$ holds for some connected component C. It can be done in polynomial time as Q is in PTIME. □

Weak lift is also weak

Proposition

If Q is PTIME quantifier, then also $\mathrm{Ram}_W(Q)$ is in PTIME.

Proof.

Check if sum of all connected components satisfies Q. □

Outline

Remind SMH...

Strong Meaning Hypothesis

Reading associated with the reciprocal in a given sentence is the strongest available reading which is consistent with relevant information supplied by the context.

SMH as instance of inferential meaning

Complexity dichotomy between strong vs. intermediate and weak interpretations of reciprocal expressions.

Does it influence our use of language?

Maybe,...

$$\mathrm{Ram}_{\mathrm{S}}(\mathrm{Q}) \Longrightarrow \mathrm{Ram}_{\mathrm{I}}(\mathrm{Q}) \Longrightarrow \mathrm{Ram}_{\mathrm{W}}(\mathrm{Q})$$

...we switch to weaker meaning when strong is too hard.

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