Semantic Complexity
A case study of collective quantification

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Outline

Motivations: why do we need semantic complexity?

Equivalent complexity thesis

Linguists and non-linguists alike agree in seeing human language as the clearest mirror we have of the activities of the human mind, and as a specially important of human culture, because it underpins most of the other components. Thus, if there is serious disagreement about whether language complexity is a universal constant or an evolving variable, that is surely a question which merits careful scrutiny. There cannot be many current topics of academic debate which have greater general human importance than this one. (Sampson, 2009)

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Existing approaches depend on implementation/theory:

- Typological approach (McWhorther, 2001; Everett, 2008)
- Information-theoretic approach (Juola, 2009)

What are the semantic bounds of everyday language?

- How to delimit ‘natural concepts’ expressible in language?
- How powerful must be our linguistic theories?
- Are some concepts harder than others?

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Logical tools to capture semantic complexity
- Semantic universals
- Definability
- Inherent complexity of the concept

Are these measures fruitfull?
- Semantic universals: CONS
- Definability
- Inherent complexity
  - Inferential meaning
  - Referential meaning

Semantic complexity of collective quantifiers
- Type-lifting strategy
- Second-order generalized quantifiers
- Definability characterization

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Generalized Quantifiers

Definition

A quantifier Q is a way of associating with each set M a function from pairs of subsets of M into $\{0, 1\}$ (False, True).

Example

$$\text{every}_M[A, B] = 1 \text{ iff } A \subseteq B$$

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$$\text{most}_M[A, B] = 1 \text{ iff } \text{card}(A \cap B) > \text{card}(A - B)$$

Space of GQs

- If $\mathrm{card}(M) = n$, then there are $2^{2^{2n}}$ GQs.
- For $n = 2$ it gives 65,536 possibilities.

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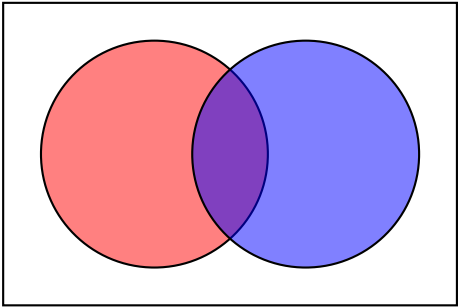
Question

Which of those correspond to simple determiners?

Isomorphism closure

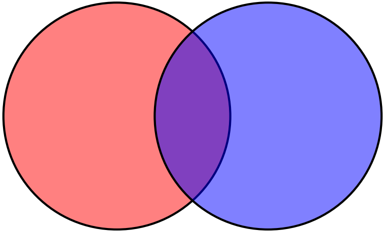
(ISOM) If $(M, A, B) \cong (M', A', B')$, then $\mathsf{Q}_{\mathbf{M}}(A, B) \Leftrightarrow \mathsf{Q}_{\mathbf{M}'}(A', B')$



Topic neutrality

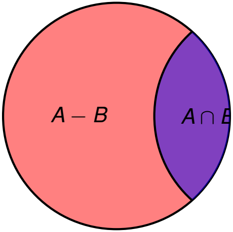
Extensionality

(EXT) If $M \subseteq M'$, then $\mathsf{Q}_{\mathbf{M}}(A, B) \Leftrightarrow \mathsf{Q}_{\mathbf{M'}}(A, B)$



Conservativity

(CONS) $\mathrm{Q}_{\mathbf{M}}(A, B) \Leftrightarrow \mathrm{Q}_{\mathbf{M}}(A, A \cap B)$



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Definability

Definition

Let Q be a generalized quantifier and $\mathcal{L}$ a logic. We say that the quantifier Q is *definable* in $\mathcal{L}$ if there is a sentence $\varphi \in \mathcal{L}$ such that for any $\mathbb{M}$:

$$\mathbb{M} \models \varphi \text{ iff } \mathsf{Q}_M[A, B].$$

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Theorem

'There exists (in)finitely many', 'most' and 'even' are not FO-definable.

Theorem (Westerståhl 1998)

In finite models, persistent CE-quantifiers are FO-definable.

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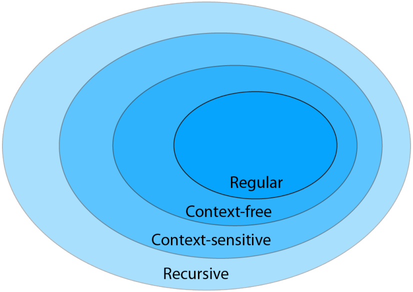
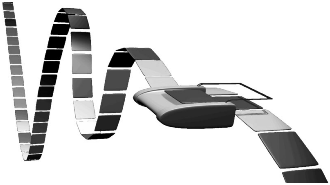
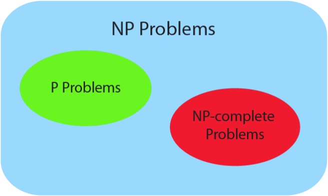
E.g. in terms of Chomsky's Hierarchy



Or (in)tractability border





Empirically adequate models (and theories) of language will give rise to NP-completeness, under an appropriate idealization to unbounded inputs. If a language model is more complex than NP, say PSPACE-hard, then our complexity thesis predicts that the system is unnaturally powerful, perhaps because it overgeneralizes from the empirical evidence or misanalyses some linguistic phenomena. (Ristad, 1993)

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Conservativity and learnability

$$\text{gleeb}_M[A, B] = 1 \text{ iff } A \not\subseteq B$$

$$\text{gleeb}'_M[A, B] = 1 \text{ iff } B \not\subseteq A$$

(a)

(b)

(Hunter & Lidz, 2010)

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FO- and HO-quantifiers

Differences in brain activity.

- Only higher-order activate working-memory capacity: recruit right dorsolateral prefrontal cortex.

(McMillan et al., 2005, Szymanik, 2007)

Outline

Various semantic problems

- Inferential meaning

  $\hookrightarrow$ complexity of reasoning (satisfiability)
  How complex are natural language arguments?
- Referential meaning

  $\hookrightarrow$ complexity of verification (model-checking)
  How hard are natural language concepts?

They are closely related (Gottlob et al., 1999).

Example of inferences

$$\frac{\text{Every Italian loves pasta and football.} \quad \text{Camilo is Italian}}{\text{Camilo loves pasta}}$$

Example of inferences

Every Italian loves pasta and football.
Camilo is Italian

Camilo loves pasta

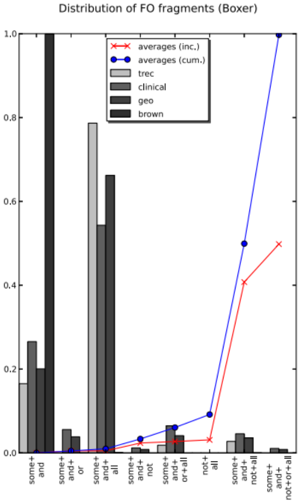
Everyone likes everyone who likes Pat
Pat doesn't like every clarinetist

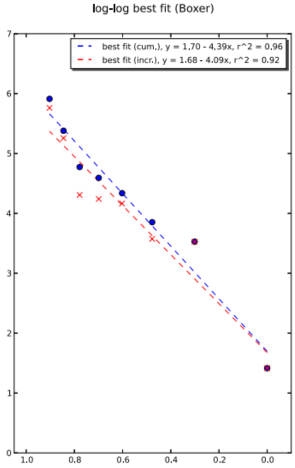
Everyone likes everyone who likes everyone who doesn't like every clarinetist

Theorem (Pratt-Hartmann 2010)

Having both negation and relatives makes fragments hard.

Principle of least effort in argumentation?





(Thorne, 2012)

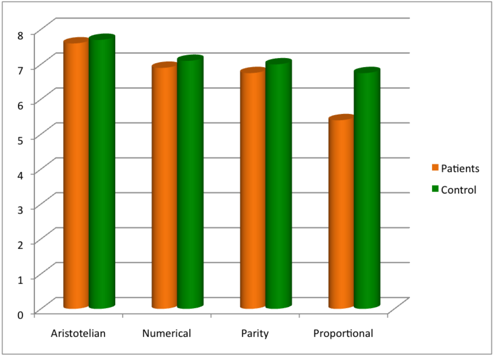
Complexity of verifying quantifiers

Quantifiers	Chomsky hierarchy
Aristotelian	REG
Numerical	REG
Parity	REG
Proportional	CFL

These classes are closed also on iterations.

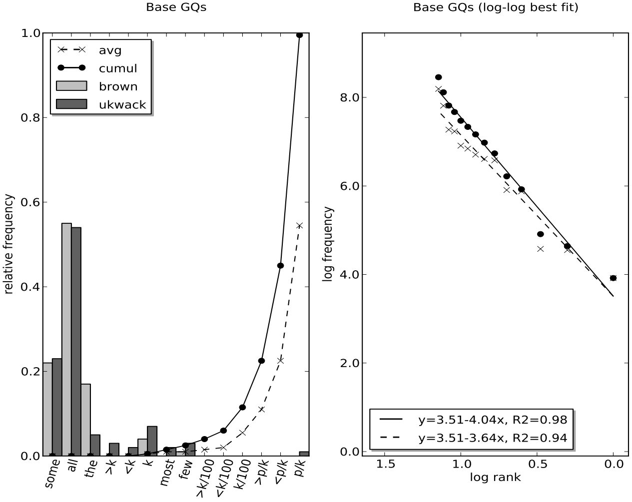
(van Benthem, 1986; Mostowski, 1998; Icard and Steinert-Threlkeld, 2013)

Processing load



(Zajenkowski, Styla, and Szymanik, 2010)

Monadic quantifier distribution and power law regression



(Thorne & Szymanik, 2014)

Intermediate summary

So far we’ve seen that various notions of complexity can lead to interesting theoretical questions as well as fruitful experiments. But how do we generalize these notions beyond distributive quantifiers?

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Collectivity

(1.) All the Knights but King Arthur *met in secret*.

(2.) Most climbers *are friends*.

(3.) John and Mary *love each other*.

(4.) The samurai *were twelve in number*.

(5.) Many girls *gathered*.

(6.) Soldiers *surrounded* the Alamo.

(7.) Tikitu and Samson *lifted* the table.

Let's start with examples

(1.) Five people lifted the table.

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(2.) Some students played poker together.

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(2.) Some students played poker together.

(2'.) $\exists X[X \subseteq \text{Students} \wedge \text{Play}(X)]$.

Existential modifier

Definition (van der Does 1992)

Fix a universe of discourse U and take any $X \subseteq U$ and $Y \subseteq \mathcal{P}(U)$. Define the existential lift Q^{EM} of a quantifier Q in the following way:

$$\mathrm{Q}^{EM}(X, Y) \text{ is true} \iff \exists Z \subseteq X[\mathrm{Q}(X, Z) \wedge Z \in Y].$$

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$$((et)((et)t)) \rightsquigarrow ((et)(((et)t)t))$$

Fact

Existential modifier arbitrarily decides invariance properties.

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Lindström quantifiers

Definition

A generalize quantifier Q is a class of models closed on isomorphism.

Examples Lindström quantifiers

$$
\begin{aligned}
\forall &= \{(A,P) \mid P = A\}. \\
\exists &= \{(A,P) \mid P \subseteq A \;\&\; P \neq \emptyset\}. \\
\text{even} &= \{(A,P) \mid P \subseteq A \;\&\; \text{card}(P) \text{ is even}\}. \\
\text{most} &= \{(A,P,S) \mid P,S \subseteq A \;\&\; \text{card}(P \cap S) > \text{card}(P - S)\}. \\
\text{M} &= \{(A,P) \mid P \subseteq A \text{ and } |P| > |A|/2\} \\
\text{some} &= \{(A,P,S) \mid P,S \subseteq A \;\&\; P \cap S \neq \emptyset\}.
\end{aligned}
$$

Second-order structures

Definition

Let $t = (s_1, \ldots, s_w)$, where $s_i = (l_1^i, \ldots, l_{r_i}^i)$ is a tuple of positive integers for $1 \leq i \leq w$. A second-order structure of type t is a structure of the form $(A, P_1, \ldots, P_w)$, where $P_i \subseteq \mathcal{P}(A^{l_1^i}) \times \cdots \times \mathcal{P}(A^{l_{r_i}^i})$.

Second-order generalized quantifiers

Definition

A second-order generalized quantifier $\mathcal{Q}$ of type t is a class of structures of type t such that $\mathcal{Q}$ is closed under isomorphisms.

Second-order GQs

$$
\begin{aligned}
\exists^2 &= \{(M,P) \mid P \subseteq \mathcal{P}(M) \ \&\ P \neq \emptyset\}. \\
\textsc{Even} &= \{(M,P) \mid P \subseteq \mathcal{P}(M) \ \&\ \mathrm{card}(P) \text{ is even}\}. \\
\textsc{Even}' &= \{(M,P) \mid P \subseteq \mathcal{P}(M) \ \&\ \forall X \in P(\mathrm{card}(X) \text{ is even})\}. \\
\textsc{Most} &= \{(M,P,S) \mid P,S \subseteq \mathcal{P}(M) \ \&\ \mathrm{card}(P \cap S) > \mathrm{card}(P - S)\}. \\
\mathrm{some}^{EM} &= \{(M,P,G) \mid P \subseteq M;\ G \subseteq \mathcal{P}(M) :\ \exists Y \subseteq P(Y \neq \emptyset \ \&\ P \in G)\}.
\end{aligned}
$$

Warning!

Do not confuse:

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- FO GQs (Lindström) with FO-definable quantifiers

  E.g. most is FO GQs but is not FO-definable.

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Do not confuse:

- FO GQs (Lindström) with FO-definable quantifiers

  E.g. most is FO GQs but is not FO-definable.
- SO GQs with SO-definable quantifiers

  E.g. MOST is SO GQs but not SO-definable.

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GQs are not enough

Theorem (Kontinen 2002)

The extension $\mathcal{L}^*$ of first-order logic by all Lindström quantifiers cannot define the monadic second-order existential quantifier, some^{EM}.

Corollary

GQs alone are not adequate for formalizing all NL quantification.

SO-definable GQs are closed on lifts

Theorem

Let Q be a Lindström quantifier definable in SO. Then Q^{EM} is definable in SO.

SO-definable GQs are closed on lifts

Theorem
Let Q *be a Lindström quantifier definable in* SO*. Then* Q^{EM} *is definable in* SO*.*

And this is the case for all SO-definable lifts:

Theorem
Let us assume that the lift $(\cdot)^*$ *and a Lindström quantifier* Q *are both definable in second-order logic. Then the collective quantifier* Q^* *is also definable in second-order logic.*

Some collectives are not definable in SO

(5.) Most groups of students played Hold'em together.

(5'.) $\textsc{Most}\, X, Y[\text{Students}(X), \text{Play}(Y)]$.

Question

Can we capture it via type-shifting?

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Theorem (Kontinen and Szymanik 2014)

The collective quantifier MOST *is not definable in second-order logic.*

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Can we capture it via type-shifting?

Theorem (Kontinen and Szymanik 2014)

The collective quantifier $\textsc{Most}$ *is not definable in second-order logic.*

Theorem (Kontinen and Szymanik 2014)

$\mathcal{Q}_1$ *is definable in* $\mathrm{SO}(\mathcal{Q}_2, +)$ *if and only if* $\mathcal{Q}_1^\star$ *is definable in* $\mathrm{FO}(\mathcal{Q}_2^\star, +, \times)$, *where:*

$$\mathcal{Q}^\star := \{\hat{\mathfrak{A}} : \mathfrak{A} \in \mathcal{Q}\},$$

where $\hat{\mathfrak{A}}$ *is the first-order encoding of* $\mathfrak{A}$.

Consequences

Corollary

The type-shifting strategy is not general enough to cover all collective quantification in natural language.

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We can have a theory of collective quantifiers similar to the one we have for distributive ones.

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Question

Have we just encountered an example where complexity restricts the expressibility of everyday language as suggested already by Ristad? And therefore, should we treat semantic complexity as another semantic universale?

Thanks for your attention!

Thanks for your attention!
And if you want to learn more details come to our course:

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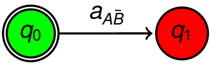
Quantifiers and Chomsky's Hierarchy

All As are B.



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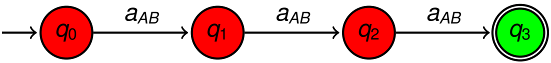

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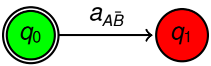

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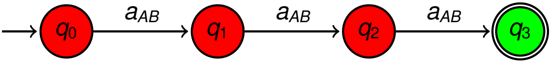

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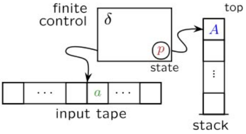

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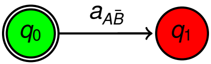

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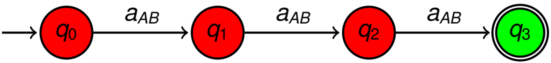
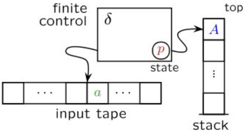


Most As are B.



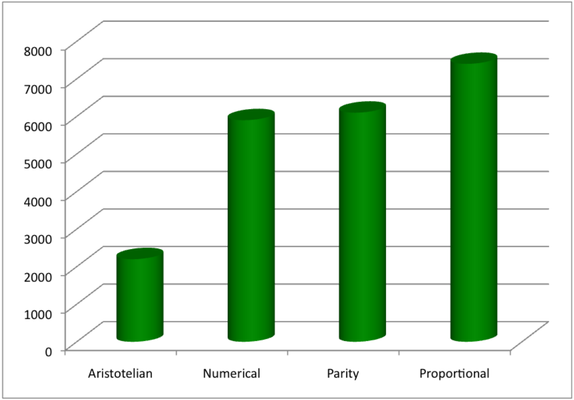
van Benthem, Essays in logical semantics, 1986

Mostowski, Computational semantics for monadic quantifiers, 1998

A simple study

More than half of the cars are yellow.

Verification times can be predicted by complexity



Szymanik & Zajenkowski, Comprehension of simple quantifiers. Empirical evaluation of a computational model, Cognitive Science, 2010

Neurobehavioral prediction wrt working memory is satisfied

Differences in brain activity.

- Only proportional quantifiers activate working-memory capacity: recruit right dorsolateral prefrontal cortex.

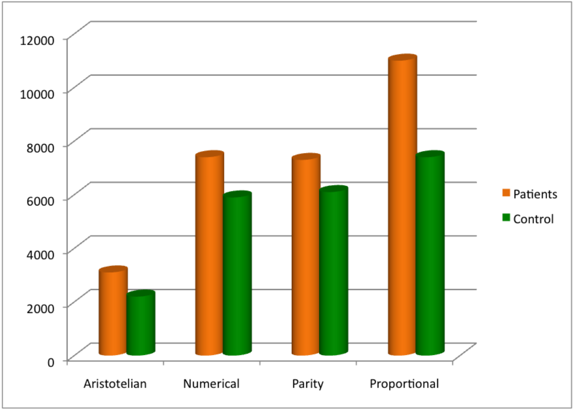
McMillan et al., Neural basis for generalized quantifiers comprehension, Neuropsychologia, 2005

Szymanik, A Note on some neuroimaging study of natural language quantifiers comprehension, Neuropsychologia, 2007

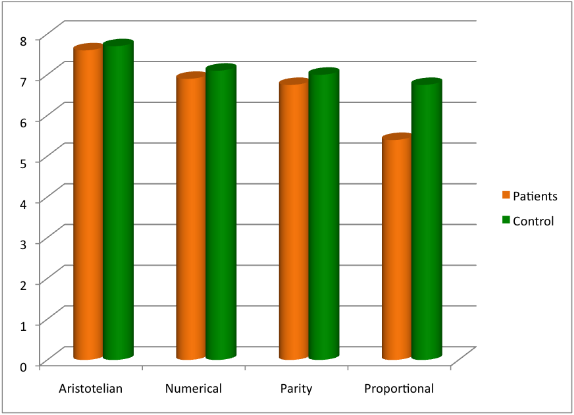
Experiment with schizophrenic patients

- Compare performance of:
  - Healthy subjects.
  - Patients with schizophrenia.
    - Known WM deficits.

Patients are generally slower



Patients are only less accurate with proportional quantifiers



Zajenkowski et al., A computational approach to quantifiers as an explanation for some language impairments in schizophrenia, Journal of Communication Disorders, 2011.

Comprehension and verification are influenced by complexity

1. Draw and verify:
   - All/Most of the dots are directly connected to each other.

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1. Draw and verify:
    - All/Most of the dots are directly connected to each other.
2. In line with complexity:
    - Fewer strong pictures for ‘most’
    - Better performance on complete graphs for ’All’-condition

Bott et al., Interpreting Tractable versus Intractable Reciprocal Sentences, Proceedings of the International Conference on Computational Semantics, 2011.

Schlotterbeck & Bott, Easy solutions for a hard problem? The computational complexity of reciprocals with quantificational antecedents, Proc. of the Logic & Cognition Workshop at ESSLLI 2012.

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Generalized Quantifiers

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A quantifier Q is a way of associating with each set M a function from pairs of subsets of M into $\{0, 1\}$ (False, True).

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$$\text{most}_M[A, B] = 1 \text{ iff } \text{card}(A \cap B) > \text{card}(A - B)$$

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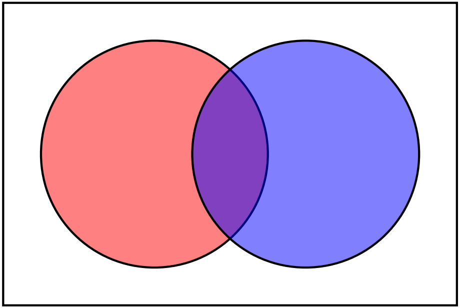
Question

Which of those correspond to simple determiners?

Isomorphism closure

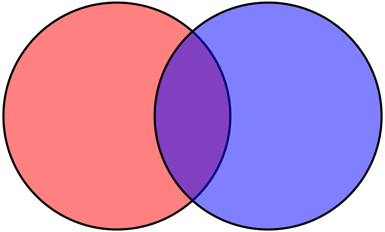
(ISOM) If $(M, A, B) \cong (M', A', B')$, then $\mathsf{Q}_{\mathbf{M}}(A, B) \Leftrightarrow \mathsf{Q}_{\mathbf{M}'}(A', B')$



Topic neutrality

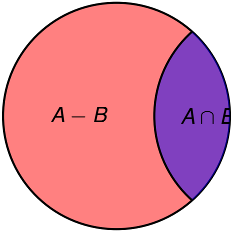
Extensionality

(EXT) If $M \subseteq M'$, then $\mathsf{Q}_{\mathbf{M}}(A, B) \Leftrightarrow \mathsf{Q}_{\mathbf{M}'}(A, B)$



Conservativity

(CONS) $\mathrm{Q}_{\mathbf{M}}(A, B) \Leftrightarrow \mathrm{Q}_{\mathbf{M}}(A, A \cap B)$



(In)tractable Reciprocal Constructions

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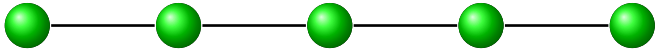
Five pitchers sat alongside each other.



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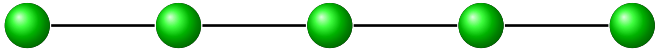

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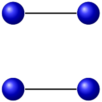

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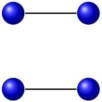

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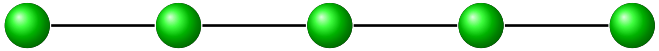

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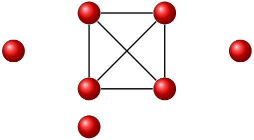

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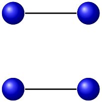

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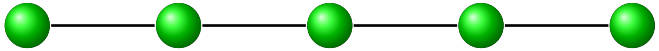
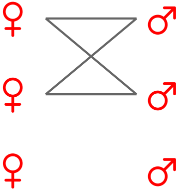


Most girls and most boys hate each other



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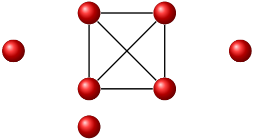
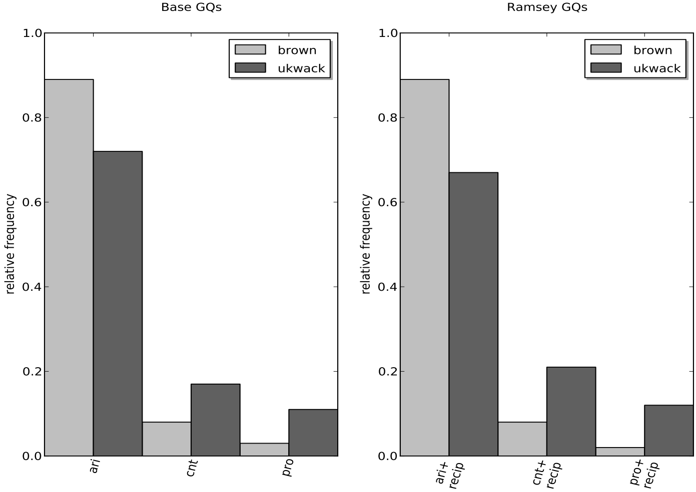


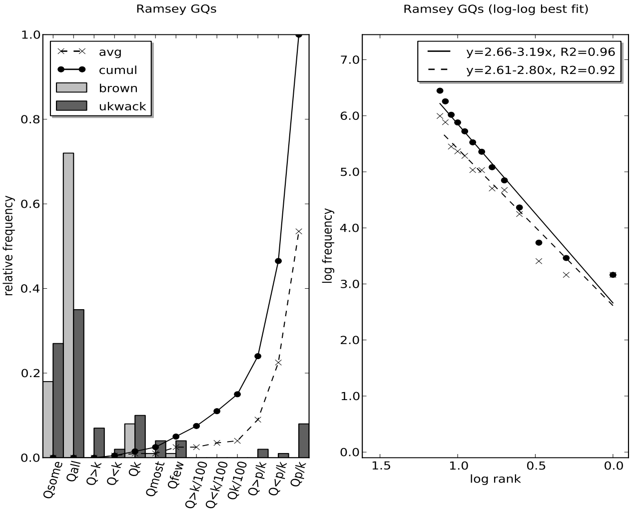
(Gierasimczuk & Szymanik, 2009; Szymanik, 2010)

Quantifier distribution by classes



(Thorne & Szymanik, 2014)

Ramsey quantifier distribution and power law regression



Summary: proof of concept

- Computationally easier expressions occur exponentially more frequent.
- Semantic complexity can quantify linguistic simplicity.
- Additional support for the cognitive studies.
- Semantic complexity is an empirically fruitful notion.
- Next step, apply it to equivalent complexity thesis.

Semantic complexity as universale

- Some expressions may be even too hard to appear in NL.
  - E.g, some collective quantifiers can be crazy complex!
- Complexity as a test of methodological plausibility of linguistic theories.

(Ristad, 1993; Mostowski & Szymanik, 2012; Kontinen & Szymanik, 2014)

Van Benthem problem

Observation

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(2.) No left-wing students met yesterday at the coffee shop.

(3.) No students met yesterday at the “Che” coffee shop.

The total number is missing

(1.) Exactly 5 students drank a whole keg of beer together.

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(1".) $\exists A \subseteq \text{Student}[\text{card}(A) = 5 \wedge \text{Drink-a-whole-keg-of-beer}(A)]$

Neutral Modifier

Definition (van der Does 1992)

Let U be a universe, $X \subseteq U$, $Y \subseteq \mathcal{P}(U)$, and Q a type (1, 1) quantifier. We define the *neutral modifier*:

$$\mathrm{Q}^{N}[X, Y] \text{ is true } \iff \mathrm{Q}\left[X, \bigcup(Y \cap \mathcal{P}(X))\right].$$

Monotonicity preservation under $(\cdot)^N$

Fact (Ben-Avi and Winter 2003)

Let Q *be a distributive determiner. If* Q *belongs to one of the classes* $\uparrow$*MON*$\uparrow$*,* $\downarrow$*MON*$\downarrow$*, MON*$\uparrow$*, MON*$\downarrow$*, then the collective determiner* Q^N *belongs to the same class. Moreover, if* Q *is conservative and* $\sim$*MON (MON*$\sim$*), then* Q^N *is also* $\sim$*MON (MON*$\sim$*).*

What about split groups?

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$$\text{card}\big(\{x | \exists A \subseteq \text{Student}[x \in A \wedge \text{Drink-a-whole-keg-of-beer}(A)]\}\big) = 5.$$

Another example . . .

Definition

We take five^{EM} to be the second-order quantifier denoting:

$$\{(M, P, G) \mid P \subseteq M;\ G \subseteq \mathcal{P}(M) :\ \exists Y \subseteq P(\mathrm{card}(Y) = 5 \ \&\ P \in G)\}.$$

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(4'.) five$^{EM}x, X[\mathrm{Student}(x), \mathrm{Lift}(X)]$.

Determiner fitting

Definition (Winter 2001)

For all $X, Y \subseteq \mathcal{P}(U)$ we have that

$$\mathrm{Q}^{\mathrm{dfit}}(X, Y) \text{ is true}$$

$$\Longleftrightarrow$$

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$$\begin{aligned}\text{card}\big(\{x \in A | A \subseteq \text{Student} \wedge \text{Drink-a-whole-keg-of-beer}(A)\}\big) = 5 \\ \wedge \exists W \subseteq \text{Student}[\text{Drink-a-whole-keg-of-beer}(W) \wedge \text{card}(W) = 5].\end{aligned}$$

It really works...

Monotonicity of Q	Monotonicity of Q^{dfit}	Example
$\uparrow$MON$\uparrow$	$\uparrow$MON$\uparrow$	Some
$\downarrow$MON$\downarrow$	$\downarrow$MON$\downarrow$	Less than five
$\downarrow$MON$\uparrow$	$\sim$MON$\uparrow$	All
$\uparrow$MON$\downarrow$	$\sim$MON$\downarrow$	Not all
$\sim$MON$\sim$	$\sim$MON$\sim$	Exactly five
$\sim$MON$\downarrow$	$\sim$MON$\downarrow$	Not all and less than five
$\sim$MON$\uparrow$	$\sim$MON$\uparrow$	Most
$\downarrow$MON$\sim$	$\sim$MON$\sim$	All or less than five
$\uparrow$MON$\sim$	$\sim$MON$\sim$	Some but not all

Table : Monotonicity under the determiner fitting operator; cf. (Ben-Avi and Winter 2003).

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- $\mathcal{L}^*$ and SO doesn't capture natural language?
- Are many-sorted (algebraic) models more plausible?
  - Type-shifting is too complex;
  - In principle this question is psychologically testable.

Σ^1_1(Ristad's)-thesis

Hypothesis

Our everyday language is semantically bounded by the properties expressible in the existential fragment of second-order logic.

Logics with Lindström quantifiers

The extension FO(Q) is defined as usual.

$$\mathfrak{A} \models Q\overline{x}_1, \ldots, \overline{x}_r \, (\phi_1(\overline{x}_1), \ldots, \phi_r(\overline{x}_r)) \text{ iff } (A, \phi_1^{\mathfrak{A}}, \ldots, \phi_r^{\mathfrak{A}}) \in Q,$$

where $\phi_i^{\mathfrak{A}} = \{\overline{a} \in A^{l_i} \mid \mathfrak{A} \models \phi_i(\overline{a})\}$

It violates invariance properties

Definition

A distributive determiner of type (1, 1) is conservative if and only if the following holds for all M and all $A, B \subseteq M$:

$$\mathrm{Q}_M[A, B] \iff Q_M[A, A \cap B].$$

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Fact

For every Q *the quantifier* Q^{EM} *is not CONS.*

...and not only because of technicalities

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We need less arbitrary approach . . .