Exploring complexity of social interactions

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Motivations

models $\longrightarrow$ computations $\longrightarrow$ cognition

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- taking computations more seriously

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$\hookrightarrow$ e.g., DEL planning

Response:

- map the feasibility borders
- identify responsible parameters
- shifting focus to concrete epistemic tasks
- from agents' perspective

$\hookrightarrow$ cf. Van Ditmarsch

Outline

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Finite Iterated Prisoners' Dilemma

	$Defect_2$	$Cooperate_2$
$Defect_1$	1, 1	4, 0
$Cooperate_1$	0, 4	3, 3

- For $N =?$ *or* $N = \infty$ rationally decide to cooperate fearing retaliation.

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Theorem (Neyman 1985)

If players have sufficiently small memory then cooperate!

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$\hookrightarrow$ c.c. considerations can vanquish some counterintuitive conclusions

$\hookrightarrow$ by modeling resource-bounded rationality

$\hookrightarrow$ linking to cognitive modeling

Question

Can c.c. provide new insights by linking economy with CogSci?

Neyman. *Bounded complexity justifies cooperation in the finitely repeated prisoners' dillema*, Economics Letters, 1985

General Complexity of Finding Equilibria

Theorem

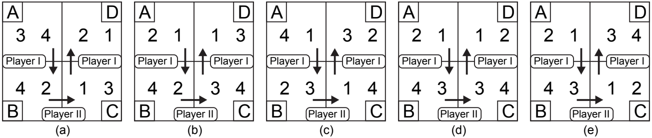
For general two-player game finding a Nash equilibrium is hard.

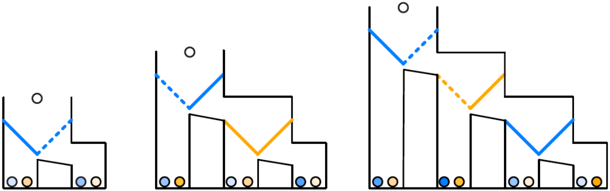
Question

- *What about interesting games?*
- *What are the factors responsible for the complexity?*

Daskalakis, Goldberg, Papadimitriou. *The complexity of computing a Nash equilibrium*, Communication ACM, 2009

Backward Induction and Higher-order Reasoning





(a) (b) (c)

Logical analysis: MDG decision trees

$$
\begin{tikzcd}
 & s,1 \arrow[dl, "l"'] \arrow[dr, "r"] & & & \\
(t1, t2) & & t,2 \arrow[dl, "l"'] \arrow[dr, "r"] & & \\
 & (s1, s2) & & u,1 \arrow[dl, "l"'] \arrow[dr, "r"] & \\
 & & (p1, p2) & & (q1, q2)
\end{tikzcd}
$$

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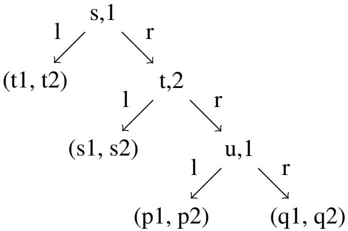

Definition

G is generic, if for each player, distinct end nodes have different pay-offs.

What's the computational complexity?

Definition

Let T be a two-player game. We define the backward induction accessibility relation on T. Let $P^T_{bi}(x, y)$ be the smallest relation on vertices of T such that:

1. $P^T_{bi}(x, x)$
2. Take $i = 1, 2$. Assume that $x \in V_i$ and $P^T_{bi}(z, y)$. If the following two conditions hold, then also $P^T_{bi}(x, y)$ holds:
   2.1 $E(x, z)$;
   2.2 there is no w, v such that $E(x, w)$, $P^T_{bi}(w, v)$, and $f_i(v) > f_i(y)$.

$$\mathbb{BI} = \{T | P^T_{bi}(s, t)\}$$

Theorem

$\mathbb{BI}$ is PTIME-complete via first-order reductions.

Szymanik. *Backward Induction is PTIME-complete*, LoRI 2013

What are the factors influencing complexity?

Definition

Let's assume that the players strictly alternate in the game. Then:

1. In a Λ_1^i tree all the nodes are controlled by Player i.
2. In a Λ_k^i tree, k-alternations, starts with an ith Player node.

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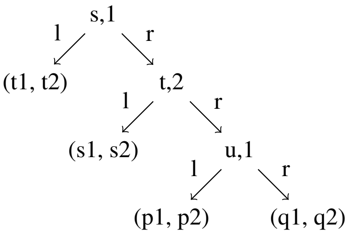


Figure : Λ_3^1 -tree

Outlook

Conjecture

For every $i, j \in \{1, 2\}$, the computational complexity of solving Λ^i_{k+1} graphs is greater than for all Λ^j_k graphs, and all Λ^i_k graphs are of the same complexity?

- $\hookrightarrow$ how higher-order reasoning links to computations?
- $\hookrightarrow$ can c.c. analysis inform cognitive models?
- $\hookrightarrow$ can c.c. help to identify rationality-obstacles?

van Benthem and Gheerbrant. Game solution, epistemic dynamics, and fixed-point logics, Fundamenta Informaticae, 2010

Szymanik, Meijering, Verbrugge. Using intrinsic complexity of turn-taking games to predict participants' reaction times, 35th CogSci, 2013

Outline

- Equilibria and Bounded Rationality
- Manipulating Information
- Epistemic Representations
- Logical Omniscience

Reasoning about information

$\Rightarrow \quad \mathbf{M} = \langle W, (R_i)_{i \in N}, V \rangle$

Case study to inquire how the complexity of various reasoning tasks is influenced by:

- choice of similarity notion for information states,
- choice of information structures,

Information Similarity

Is everyone in the same state of mind in both situations?

Theorem

1. *Kripke model isomorphism is 'hard'.*
2. *Multi-agent epistemic S5 model bisimilarity is P-complete*

Inducing Information Similarity

Is it possible to give Lucy info that she's in the same state of mind as Shroeder?

Theorem

1. *For arbitrary Kripke models: NP-hard.*
2. *For S5: in linear time.*

van Ditmrsch & French. Simulation and Information: Quantifying over epistemic events , KRAMAS, 2008

Classification Problem

Problem	Tractable?	Comments
Kripke model isomorphism	unknown	in GI
Epistemic model bisimilarity	Yes	?? P-hard for $\geq$ 2 agents
Flipped horizon bisimilarity	Yes	P-complete for arbitrary models
Kripke submodel bisimulation	No	NP-complete for arbitrary models; in linear time for S5
Local S5 submodel bisimulation	1 agent: Yes	unknown
Total S5 submodel bisimulation	1 agent: Yes	?? NP-complete for $\geq$ 2 agents
Kripke submod. simulation (equiv.)	No	?? in P for single agent S5

?? indicates a conjecture

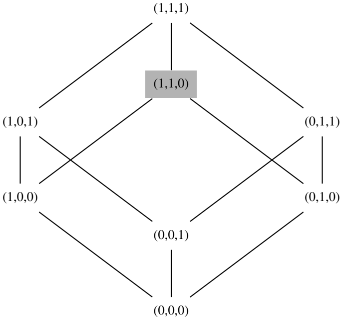
Dégremont, Kurzen, Szymanik Exploring the Tractability Border in Epistemic Tasks , Synthese, 2012

Outline

Muddy Children

You are visiting a relative, who has three children. While you are having coffee in the living-room, the kids are playing outside. When they come back home, their father says: (1) 'At least one of you has mud on your forehead'. Then, he asks the children: (I) 'Can you tell for sure whether you have mud on your forehead? If yes, announce your status'. Children know that their father never lies and that they are all perfect logical reasoners. Each child can see the mud on others but cannot see his or her own forehead. Nothing happens. But after the father repeats the question for the second time suddenly all muddy children know that they have mud on their forehead. How is that possible?

Epistemic Logic Representation



More Succinct Representations

Observation

The scenario has two types of agents. Every clean child's observation is quantitatively equivalent to the observation of all other clean children. Similarly, every muddy child observes the same as all other muddy children.

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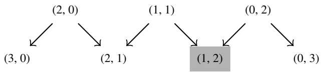

- $\hookrightarrow$ concise modeling of concrete epistemic scenarios
- $\hookrightarrow$ agent's internal representation

Gierasimczuk & Szymanik. *A note on a generalization of the Muddy Children Puzzle*, TARK 2011

Wang. *Epistemic Modelling and Protocol Dynamics* , PhD ILLC 2010

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Agents' knowledge closed under deduction $\hookrightarrow$ logical omniscience

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> *The view that machines cannot give rise to surprises is due, I believe, to a fallacy to which philosophers and mathematicians are particularly subject. This is the assumption that as soon as a fact is presented to a mind all consequences of that fact spring into the mind simultaneously with it. It is a very useful assumption under many circumstances, but one too easily forgets that it is false.—Turing[1950]*

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Question

Can we give some formal account of 'knowledge' able to accommodate people learning new things without leaving their armchairs?—Hintikka[1962]

Knowing Prime Numbers

The largest known prime number is $p = 2^{57885161} - 1$

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Why not? I'd say as we don't know any efficient algorithm that outputs p'.

Procedural Knowledge

Internal algorithm by which you can efficiently answer a large (infinite?) set of questions in some form

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Internal algorithm by which you can efficiently answer a large (infinite?) set of questions in some form

Example

1. Do you know Dutch?
2. Do you know Texas Hold'em?
3. Do you know calculus?

Aaronson. *Why Philosophers Should Care About Computational Complexity*, 2012

How does it help with logical omniscience?

Question

Can we give axiomatics for 'knowing how to compute efficiently'?

Example

If you know how to efficiently compute f and g, then you also efficiently know $f + g$

Cobham Axioms for FP

Theorem

FP *is the smallest class satisfying:*

1. *Every constant f. and every f. that is nonzero only finitely many times is in* FP
2. *If* $f(x), g(x) \in$ FP *then* $< f(x), g(x) > \in$ FP
3. *If* $f(x), g(x) \in$ FP *then* $f(g(x)) \in$ FP
4. $+, \times \in$ FP
5. $|x|$, Π_1, Π_2, $bit(< x, i >)$, $diff(< x, i >) \in$ FP
6. $2^{|x|^2}$
7. *If* $f(x) \in$ FP, *and for all* $x \in \mathbb{N}$, $|f(x)| \leq |x|$, *then the function*

$$g(< x, k >) = \begin{cases} f(g(< x, \lfloor k/2 \rfloor)) & \text{if } k > 1 \\ x & \text{if } k = 1. \end{cases} \in \text{FP}$$

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Theorem (Leivant 1994)

$f \in \mathsf{FP}$ *iff* f *is computed by a program that can be proved correct in* SO *with comprehension restricted to positive quantifier-free formulas.*

Example

Do you know answer to the following questions:

1. Is $1591 = 43 \times 37$?
2. What are the prime factors of 1591?

Knowing *How* and Knowing *That*

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- $\hookrightarrow$ knowledge := agents' question answering capacities
- $\hookrightarrow$ for infinite sets of related questions
- $\hookrightarrow$ linking to procedural perspective in NL semantics, and
- $\hookrightarrow$ learnability take on 'knowledge'

Stalnaker. *The problem of logical omniscience, I and II*, 1999

Summing up

Complexity considerations may bring our models closer to cognition

http://jakubszymanik.com/false-belief/