From Logic to Behavior

Modern semantics and complexity theory in cognitive modeling

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Outline

Divide between logic and psychology

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$\hookrightarrow$ interpretation and processing

Modern logic should be a part of CogSci toolbox

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- Logical engine of Bayesian modeling

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Expensive experiments and messy computational models should be built upon more principled foundational approach.

Evaluating cognitive models

Along the following dimensions:

- logical relationships, e.g., incompatibility or identity;
- explanatory power, e.g., what can be expressed;
- computational plausibility, e.g., tractability.

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Information processing and 3 levels of Marr

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   - problems that a cognitive ability has to overcome
2. Algorithmic level:
   - the algorithms that are used to achieve a solution
   - compute f
3. Implementation level:
   - how this is actually done in neural activity

Marr. *Vision: a computational investigation into the human representation and processing visual information*, 1983

Extending levels of explanation in CogSci

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Observation
Logical analysis informs about intrinsic properties of a problem.

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↪ Level 1.5: using logic to predict behavior!

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> *There is nothing as practical as good theory.*
>
> *(Lewin, 1951)*

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Example (False belief tasks)

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Lambalgen & Stenning. *Human reasoning and cognitive science*, 2008

Braüner. Hybrid-Logical Reasoning in False-Belief Tasks, TARK 2013

Van Ditmarsch & Labuschagne. My Beliefs about Your Beliefs, Synthese 2007

Level 1.5: from formalization to actual reasoning

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Example (Using proof-theory)

- Monotonicity calculus as processing model for syllogistic.
- Shorter proof = simpler syllogism.

Geurts. Reasoning with quantifiers, Cognition, 2003

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- Analytic tableaux for MasterMind game.
- Simpler proof = simpler game.

Gierasimczuk et al. Logical and psychological analysis of Mastermind, J. of Logic, Language, and Information, 2013

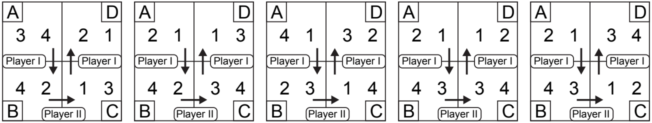
Outline

Level 1.5: more semantic approach

- To capture structural properties of the task
- Independent from particular formalization

Turn-based games

Turn-based games



(a) (b) (c) (d) (e)

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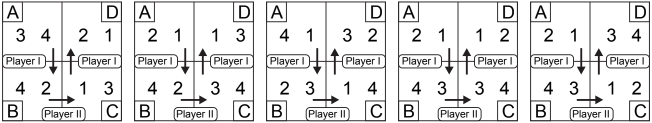

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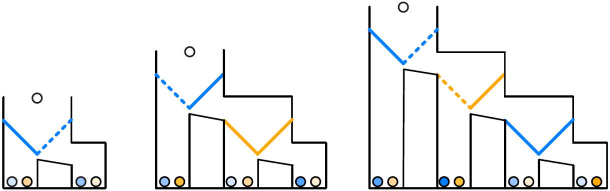
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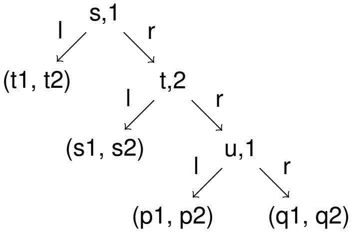
MDG decision trees



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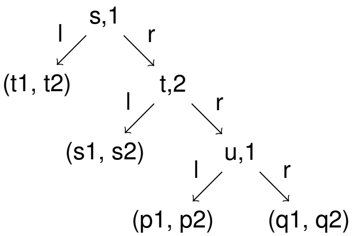

Definition

G is generic, if for each player, distinct end nodes have different pay-offs.

Question

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What are the cognitively important structural properties?

Alternation type

Definition

Let's assume that the players strictly alternate in the game. Then:

1. In a Λ_1^i tree all the nodes are controlled by Player i.
2. In a Λ_k^i tree, k-alternations, starts with an ith Player node.

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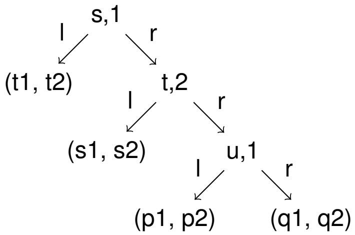

Figure : Λ^1_3 -tree

Pay-off structure

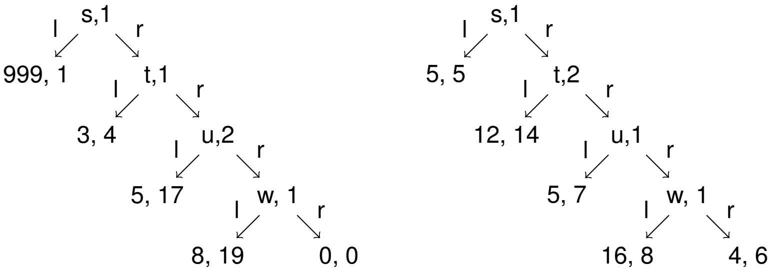

Figure : Two Λ_3^1 trees.

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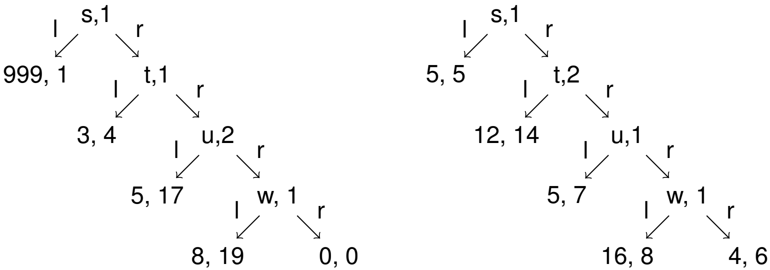


Figure : Two Λ_3^1 trees.

Forward reasoning + backtracking

T^--example

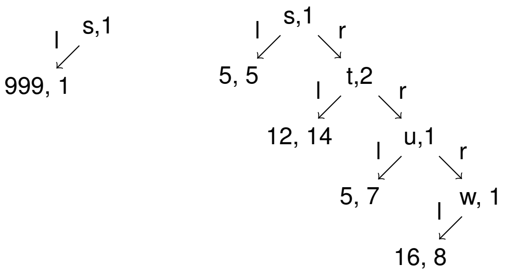


Figure : Λ_1^1 tree and Λ_3^1 tree

T^-

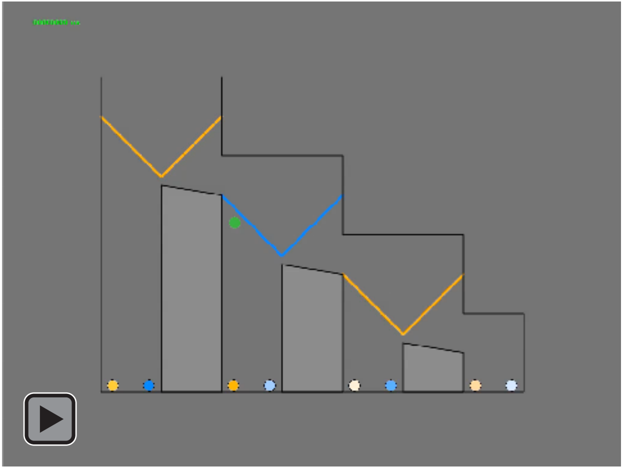
Definition

If T is a generic game tree with the root node controlled by Player 1 (2) and n is the highest pay-off for Player 1 (2), then T^- is the minimal subtree of T containing the root node and the node with pay-off n for Player 1 (2).

Conjecture

Let us take two MDG trials T_1 and T_2. T_1 is easier for participants than T_2 if and only if T_1^- is lower in the tree alternation hierarchy than T_2^-.

Experiment



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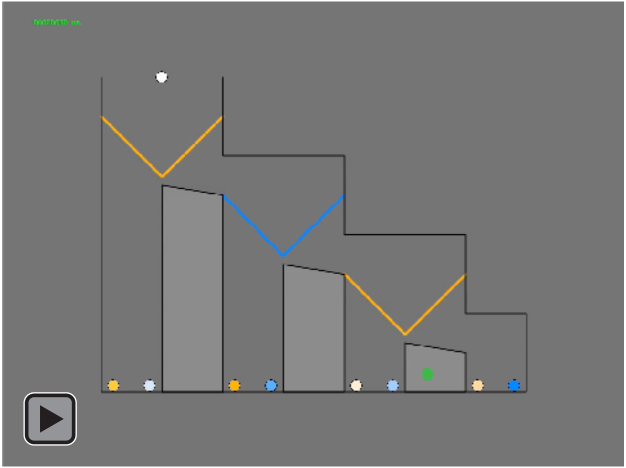

Results

- Structural properties responsible for the cognitive difficulty
- Results generalized to other turn-based games
- FRB avoids higher-order reasoning

Szymanik et al.. Using intrinsic complexity of turn-taking games to predict participantsÕ reaction times, CogSci 2013

Outline

Practical computability

$$f : \text{initial state} \longrightarrow \text{desired state}$$

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- computational resource constraints;
- realistic time and memory;
- bounded agency

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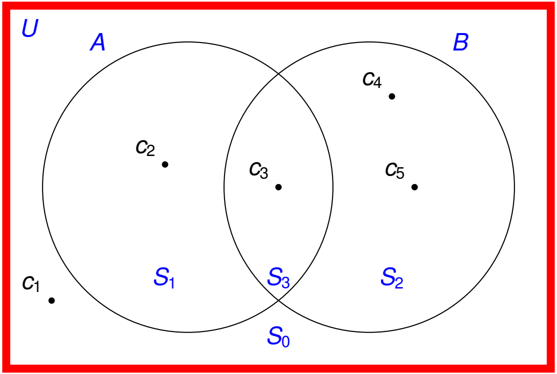
$\hookrightarrow$ Level 1.5: computational properties

Simple sentences

1. All poets have low self-esteem.
2. Some dean danced nude on the table.
3. At least 3 grad students prepared presentations.
4. An even number of the students saw a ghost.
5. Most of the students think they are smart.
6. Less than half of the students received good marks.
7. Many of the soldiers have not eaten for several days.
8. A few of the conservatives complained about taxes.

Corresponding structures



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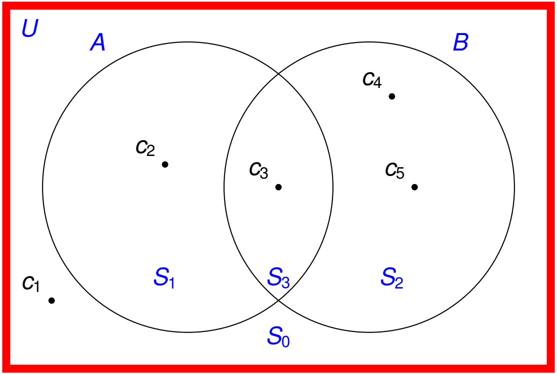
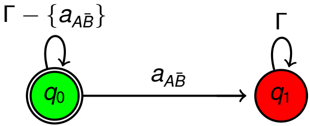


. . . and corresponding computations

Aristotelian quantifiers

"all", "some", "no", and "not all"

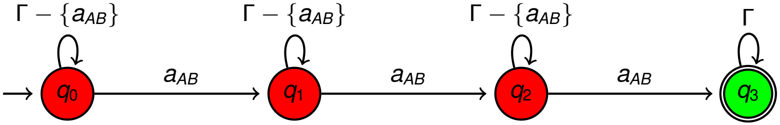


Finite automaton recognizing L_{All}

$$L_{\mathrm{All}} = \{\alpha \in \Gamma^* : \#a_{A\bar{B}}(\alpha) = 0\}$$

Cardinal quantifiers

E.g. “more than 2”, “less than 7”, and “between 8 and 11”

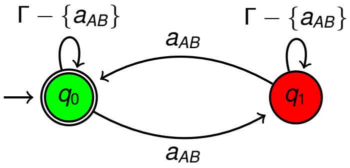


Finite automaton recognizing $L_{\text{More than two}}$

$$L_{\text{More than two}} = \{\alpha \in \Gamma^* : \#a_{AB}(\alpha) > 2\}$$

Parity quantifiers

E.g. "an even number", "an odd number"

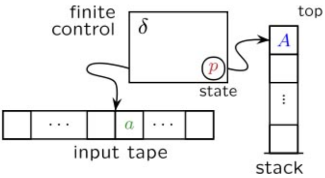


Finite automaton recognizing L_{Even}

$$L_{\text{Even}} = \{\alpha \in \Gamma^* : \#a_{AB}(\alpha) \text{ is even}\}$$

Proportional quantifiers

- E.g. "most", "less than half".
- Most *As are B* iff $\text{card}(A \cap B) > \text{card}(A - B)$.
- $L_{\text{Most}} = \{\alpha \in \Gamma^* : \#a_{AB}(\alpha) > \#a_{A\bar{B}}(\alpha)\}$.
- There is no finite automaton recognizing this language.
- We need internal memory.
- A push-down automata will do.



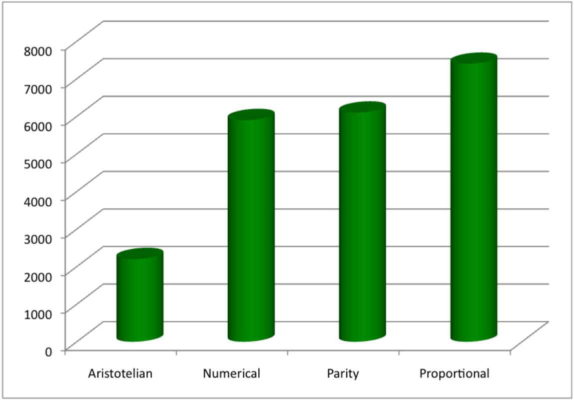
Does it say anything about processing?

Question

Do minimal automata predict differences in verification?

A simple study

More than half of the cars are yellow.



Szymaniki & Zajenkowski, Comprehension of simple quantifiers. Empirical evaluation of a computational model, Cognitive Science, 2010

Neurobehavioral studies

Differences in brain activity.

- All quantifiers are associated with numerosity: recruit right inferior parietal cortex.
- Only higher-order activate working-memory capacity: recruit right dorsolateral prefrontal cortex.

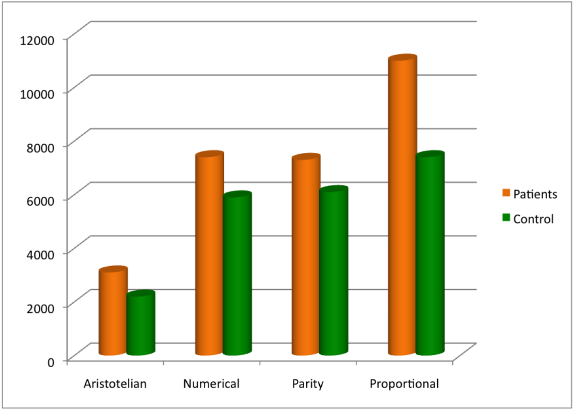
McMillan et al., Neural basis for generalized quantifiers comprehension, Neuropsychologia, 2005

Szymanik, A Note on some neuroimaging study of natural language quantifiers comprehension, Neuropsychologia, 2007

Experiment with schizophrenic patients

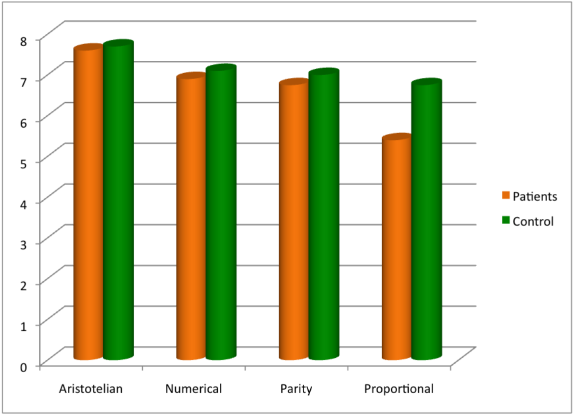
- Compare performance of:
  - Healthy subjects.
  - Patients with schizophrenia.
    - Known WM deficits.

RT data



12000
10000
8000
6000
4000
2000
0
Aristotelian
Numerical
Parity
Proportional
Patients
Control

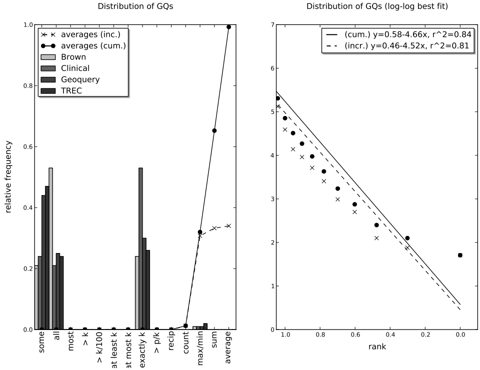
Accuracy data



Zajenkowski et al., A computational approach to quantifiers as an explanation for some language impairments in schizophrenia, Journal of Communication Disorders, 2011.

Quantifier distribution in language

Distribution is skewed towards quantifiers of low complexity.



Thorne & Szymanik. Generalized Quantifier Distribution and Semantic Complexity, 2013.

Outline

Summary

- Computational awareness in logic of agency and semantics $\hookrightarrow$ CogSci.

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- Computational awareness in logic of agency and semantics ↪ CogSci.
- Radically beyond psychology of reasoning:
  ↪ focusing on cognitive processes rather than on logical correctness
  ↪ computational turn calls for sophisticated experiments
  ↪ collaboration is needed more than ever!

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- It revolves around: interpretation, information, and computation
- It helps predict behavior
- Logical perspective extends the notion of explanation in CogSci
- $\hookrightarrow$ Level 1.5

More examples and discussion

- Isaac, Szymanik, and Verbrugge. Logic and Complexity in Cognitive Science, *Johan van Benthem on Logical and Informational Dynamics*, Trends in Logic, Outstanding Contributions book series, Springer 2013
- Szymanik and Verbrugge (Eds). Special issue of *Journal of Logic, Language, and Information on 'Logic and Cognition'*

↪ websites of various courses