Logic and Cognition

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Logic and Philosophy Today
Delhi, January 6, 2011

Outline

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Goals of the talk

Discuss examples of:

Goals of the talk

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1. Logics motivated by CogSci;

Goals of the talk

Discuss examples of:

1. Logics motivated by CogSci;
2. Experiments motivated by logics;

Goals of the talk

Discuss examples of:

1. Logics motivated by CogSci;
2. Experiments motivated by logics;
3. Selection of experimental methods.

How can logic contribute?

1. In building cognitive theories;

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1. In building cognitive theories;
2. In computational modeling;

How can logic contribute?

1. In building cognitive theories;
2. In computational modeling;
3. In designing experiments.

Classically 3 levels of Marr

1. Computational level:
    - specify cognitive task:
        - f: initial state $\longrightarrow$ desired state
    - problems that a cognitive ability has to overcome

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   - problems that a cognitive ability has to overcome
2. Algorithmic level:
   - the algorithms that may be used to achieve a solution
   - compute f
3. Implementation level:
   - how this is actually done in neural activity

Marr, Vision: a computational investigation into the human representation and processing visual information, 1983

Extending Classical Perspective

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Observation

Logical analysis informs about intrinsic properties of a problem.

Extending Classical Perspective

Observation

Logical analysis informs about intrinsic properties of a problem.

$\hookrightarrow$ Level 1.5

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Mastermind

The game consists of:

- a decoding board;
- code pegs of n colours;
- key (feedback) pegs (black and white).

Players:

- The codemaker: chooses a secret pattern.
- The codebreaker: guesses the pattern.

Mastermind

Moves:

- Each guess: placing a row of code pegs.
- The codemaker provides feedback.
  - Black key for each code peg of correct color and position.
  - White key for each peg of correct color but wrong position.
- After that another guess is made.

Winning conditions for k rounds:

- The codebreaker: obtains the solution within k rounds.
- The codemaker: otherwise.

Mastermind: an inductive inquiry game

- Trials of experimentation and evaluation.
- Interactive game.
- How to transform it into a reasoning task?

MM Setting in Rekentuin

MM Setting in Rekentuin

MM in Rekentuin

- massive data bank (over 150 schools in The Netherlands);
- the next step: a logical reasoning system;
- perhaps similar to the one for syllogisms.

Gierasimczuk et al., Static Mastermind in Rekentuin. A computational, logical, and cognitive perspective, under construction

Syllogistic Reasoning: Meta-data analysis

Table 1
Percentage of times each syllogistic conclusion was endorsed according to the meta-analysis by Chater and Oaksford (1999)[a]

premisses & figure	A	I	E	O	premisses & figure	A	I	E	O	premisses & figure	A	I	E	O
AA1	90	5	0	0	AO1	1	6	1	57	IO1	3	4	1	30
AA2	58	8	1	1	AO2	0	6	3	67	IO2	1	5	4	37
AA3	57	29	0	0	AO3	0	10	0	66	IO3	0	9	1	29
AA4	75	16	1	1	AO4	0	5	3	72	IO4	0	5	1	44
AI1	0	92	3	3	OA1	0	3	3	68	OI1	4	6	0	35
AI2	0	57	3	11	OA2	0	11	5	56	OI2	0	8	3	35
AI3	1	89	1	3	OA3	0	15	3	69	OI3	1	9	1	31
AI4	0	71	0	1	OA4	1	3	6	27	OI4	3	8	2	29
IA1	0	72	0	6	II1	0	41	3	4	EE1	0	1	34	1
IA2	13	49	3	12	II2	1	42	3	3	EE2	3	3	14	3
IA3	2	85	1	4	II3	0	24	3	1	EE3	0	0	18	3
IA4	0	91	1	1	II4	0	42	0	1	EE4	0	3	31	1
AE1	0	3	59	6	IE1	1	1	22	16	EO1	1	8	8	23
AE2	0	0	88	1	IE2	0	0	39	30	EO2	0	13	7	11
AE3	0	1	61	13	IE3	0	1	30	33	EO3	0	0	9	28
AE4	0	3	87	2	IE4	0	1	28	44	EO4	0	5	8	12
EA1	0	1	87	3	EI1	0	5	15	66	OE1	1	0	14	5
EA2	0	0	89	3	EI2	1	1	21	52	OE2	0	8	11	16
EA3	0	0	64	22	EI3	0	6	15	48	OE3	0	5	12	18
EA4	1	3	61	8	EI4	0	2	32	27	OE4	0	19	9	14
										OO1	1	8	1	22
										OO2	0	16	5	10
										OO3	1	6	0	15
										OO4	1	4	1	25

A = all E = no
I = some O = some … not

[a] All figures have been rounded to the nearest integer; valid conclusions are shaded. Whenever two conclusions in the same row are valid, only the first one is valid in predicate logic.

Chater and Oaksford, The probability heuristic model of syllogistic reasoning, Cognitive Psychology, 1999

Monotonicity calculus

- Logic rendering many valid arguments.
- Including syllogistic.
- Pivoting on monotonicity, e.g.,

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$$
\begin{array}{cccc}
\text{Rule 1} & \text{Rule 2} & \text{Example 1} & \text{Example 2} \\
\alpha \implies \beta & \beta \implies \alpha & \mathit{all}(A, B) & \mathit{all}(C, B) \\
\ldots \alpha^{+} \ldots & \ldots \alpha^{-} \ldots & \mathit{some}(A^{+}, C) & \mathit{no}(B^{-}, A) \\
\hline
\ldots \beta^{+} \ldots & \ldots \beta^{-} \ldots & \mathit{some}(B^{+}, C) & \mathit{no}(C^{-}, A)
\end{array}
$$

Monotonicity calculus

- Logic rendering many valid arguments.
- Including syllogistic.
- Pivoting on monotonicity, e.g.,

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\hline
\ldots \beta^{+} \ldots & \ldots \beta^{-} \ldots & \textit{some}(B^{+}, C) & \textit{no}(C^{-}, A)
\end{array}$$

$$\begin{array}{cc}
\text{Conversion} & \text{No/All-not} \\
\mathrm{Q}(A, B) & \mathrm{no(A, B)} \\
\hline
\mathrm{Q}(B, A),\ \mathrm{Q} = \textit{some} & \textit{all}(A, \textit{not } B)
\end{array}$$

Processing model: example

Example

1. $no(B, C)$ premiss
2. $some(B, A)$ premiss

Processing model: example

Example

1. *no*(B, C) premiss
2. *some*(B, A) premiss
3. *some*(A, B^+) Conv from 2

Processing model: example

Example

1. $no(B, C)$ premiss
2. $some(B, A)$ premiss
3. $some(A, B^+)$ Conv from 2
4. $all(B, not\ C)$ No/All-not from 1

Processing model: example

Example

1. $no(B, C)$ premiss
2. $some(B, A)$ premiss
3. $some(A, B^{+})$ Conv from 2
4. $all(B, not\ C)$ No/All-not from 1
5. $some(A, not\ C)$ Mon from 3 and 4

Processing model: example

1. The shorter the proof the easier the syllogism.

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   - Level 1.5,

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Processing model: example

1. The shorter the proof the easier the syllogism.
   - Level 1.5,
   - Rule application may be empirically weighted,
2. It gives a good fit with data.

Table 4
Predicted difficulty of valid syllogisms according to the model described in the text, compared with Chater and Oaksford's scores (in parentheses)

AA1A	80	(90)	OA3O	70	(69)	EA1O	40	(3)
EA1E	80	(87)	AO2O	70	(67)	EA2O	40	(3)
EA2E	80	(89)	EI1O	60	(66)	EA3O	40	(22)
AE2E	80	(88)	EI2O	60	(52)	EA4O	40	(8)
AE4E	80	(87)	EI3O	60	(48)	AE2O	40	(1)
IA3I	80	(85)	EI4O	60	(27)	AE4O	40	(2)
IA4I	80	(91)	AA1I	60	(5)			
AI1I	80	(92)	AA3I	60	(29)			
AI3I	80	(89)	AA4I	60	(16)			

Geurts, Reasoning with quantifiers, Cognition, 2003

Outline

Complexity of quantifiers

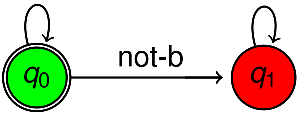
Definability	Examples	Recognized by
FO	"all", "at least 3"	acyclic FA
FO(D_n)	"an even number"	FA
PrA	"most", "less than half"	PDA

Quantifiers, definability, and complexity of automata

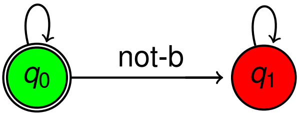
Van Benthem, Essays in logical semantics, 1986.

Mostowski, Computational semantics for monadic quantifiers, 1998.

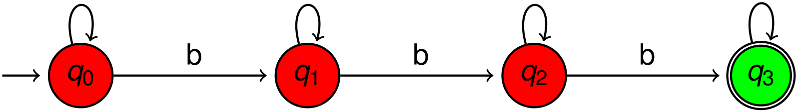
All flowers are blue.



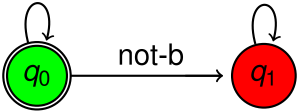
All flowers are blue.



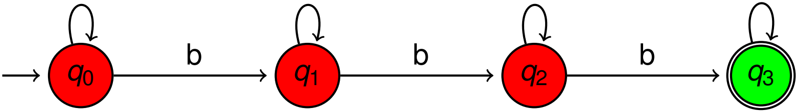
At least 3 flowers are blue.



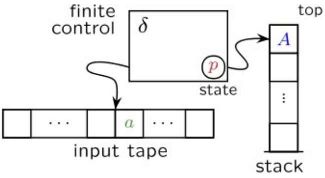
All flowers are blue.



At least 3 flowers are blue.



Most of the flowers are blue.

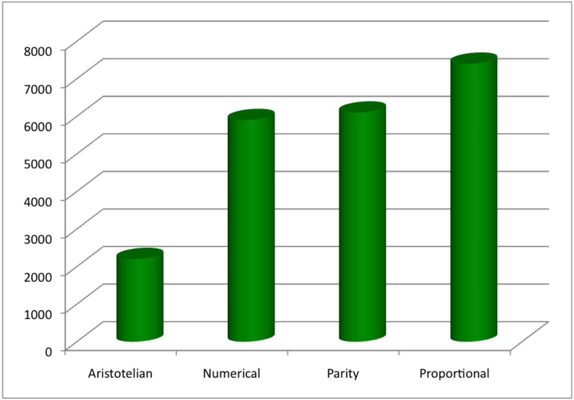


Does it say anything about processing?

Question

Do minimal automata predict differences in verification?

Complexity and reaction time



Szymaniki & Zajenkowski, Comprehension of simple quantifiers. Empirical evaluation of a computational model, Cognitive Science, 2010

Complexity and working memory

- Compare performance of:

Complexity and working memory

- Compare performance of:
  - Healthy subjects.

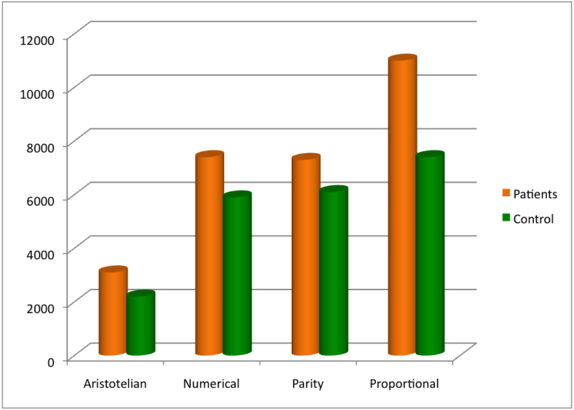
Complexity and working memory

- Compare performance of:
  - Healthy subjects.
  - Patients with schizophrenia.

Complexity and working memory

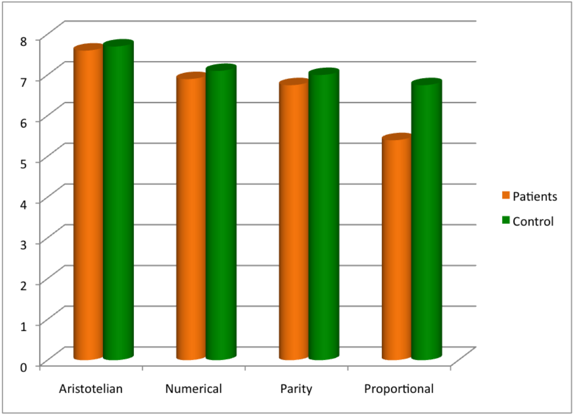
- Compare performance of:
  - Healthy subjects.
  - Patients with schizophrenia.
    - Known working memory deficits.

RT data



12000
10000
8000
6000
4000
2000
0
Aristotelian
Numerical
Parity
Proportional
Patients
Control

Accuracy data



Zajenkowski et al., A computational approach to quantifiers as an explanation for some language impairments in schizophrenia, under review.

Tractability/Intractability:

extending difficulty/complexity analogy

1. Most villagers and most townsmen hate each other.
2. All/Most of the dots are connected to each other.

Conjecture

Subjects avoid intractable interpretations.

Gierasimczuk and Szymanik, Branching quantification vs. two-way quantification, Journal of Semantics, 2009

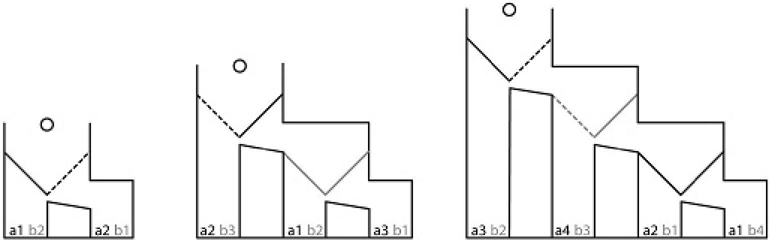
Szymanik, Computational Complexity of Polyadic Lifts of Generalized Quantifiers in Natural Language, Linguistics and Philosophy, 2010.

Bott et al., Interpreting Tractable versus Intractable Reciprocal Sentences, Proceedings of the International Conference on Computational Semantics, 2011.

Outline

Higher-order social reasoning

Marble Drop Game



Question

- Subjects are good in second-order reasonings (Mean Acc =0,91; Mean RT=7.8).
- And they even get better with training.

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Question

How are they doing it? Do they apply backward induction?

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- And they even get better with training.

Question

How are they doing it? Do they apply backward induction?

Question

How can we try to answer the question?

Method

- Registering subjects' behavior.

Method

- Registering subjects' behavior.
- Tracking eye fixations.

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- Using BI suggests fixation pattern:
  - Bin 4, 3, next Bin 2, finally Bin 1.

Method

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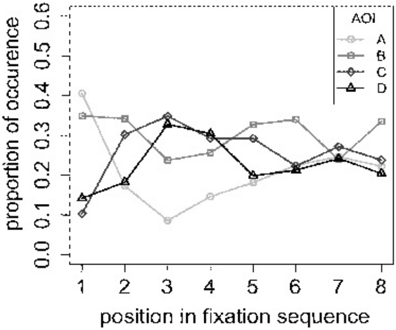
- Tracking eye fixations.
- Using BI suggests fixation pattern:
  - Bin 4, 3, next Bin 2, finally Bin 1.
- Area of Interests pattern for BI:
  - 4321
  - 3421

Results



Data consistent with AOIs: 1234 against BI hypothesis!

Meijering et al., Context facilitates theory of mind: What eye movements tell about higher-order strategic reasoning, 2011.

Outline

McMillan et al. fMRI studies

Differences in brain activity.

McMillan et al. fMRI studies

Differences in brain activity.

- All quantifiers are associated with numerosity: recruit right inferior parietal cortex.
- Only higher-order activate working-memory capacity: recruit right dorsolateral prefrontal cortex.

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Differences in brain activity.

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  recruit right inferior parietal cortex.
- Only higher-order activate working-memory capacity:
  recruit right dorsolateral prefrontal cortex.

McMillan et al., Neural basis for generalized quantifiers comprehension, 2005

Szymanik, A Note on some neuroimaging study of natural language quantifiers comprehension, Neuropsychologia, 2007

Szymanik & Zajenkowski, Quantifiers and working memory, LNCS, 2010

Outline

To sum up

Level 1.5	Mastermind, Syllogisms, Verification
Level 2	Marble Drop Game
Level 3	Quantifiers and Definability

Discussion

- Adequacy of Marr's Levels.
- Idealized logical agents.
- How to measure difficulty?
- Logic & CogSci: can the benefits be mutual?

धन्यवाद

Static Mastermind (Chvatal 1983)

- finding the minimum number of guesses the codebreaker can make all at once at the beginning of the game;
- without waiting for the answers;
- and upon receiving the answers;
- completely determine the code in the next guess.

Observation (Greenwell 1999)

Static Mastermind ($n = 6, \ell = 4$) can be solved with six initial guesses. In particular: (1, 2, 2, 1), (2, 3, 5, 4), (3, 3, 1, 1), (4, 5, 2, 4), (5, 6, 5, 6), (6, 6, 4, 3).

Conjecture

It is not possible to reduce to five (exhaustive check: approx 3.7×10^{15} computations).

Static Mastermind: Computational Complexity

Mastermind (satisfiability) decision problem:

Input A set of guesses G and their corresponding scores.

Question Is there at least one valid solution?

Theorem

Mastermind Problem in NP-complete wrt ℓ (positions).

Objective computational measure!

Monotonicity profiles determine difficulty

1. Some of the sopranos sang with more than three of the tenors.
2. None of the sopranos sang with fewer than three of the tenors.
3. Some of the sopranos sang with fewer than three of the tenors.

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$$\frac{\begin{array}{c}\mathrm{Q}_1\,A \text{ played against } \mathrm{Q}_2\,B\\ \text{All B were C.}\end{array}}{\mathrm{Q}_1\,A \text{ played against } \mathrm{Q}_2\,C}$$

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$$\uparrow \mathrm{Q}_1 \uparrow \mathrm{Q}_2 < \downarrow \mathrm{Q}_1 \downarrow \mathrm{Q}_2 < \begin{array}{c} \uparrow \mathrm{Q}_1 \downarrow \mathrm{Q}_2 \\ \downarrow \mathrm{Q}_1 \uparrow \mathrm{Q}_2 \end{array}$$

Geurts and Van der Slik, Monotonicity and Processing Load, Journal of Semantics, 2005

Discussion

Conclusion

Automata model is psychologically plausible.

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Computational complexity $\approx$ cognitive difficulty.

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Computational complexity $\approx$ *cognitive difficulty.*

- As far as we know this is the first empirical proof.

Discussion

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Automata model is psychologically plausible.

Conclusion

Computational complexity $\approx$ cognitive difficulty.

- As far as we know this is the first empirical proof.
- Between Marr's level 1 and 2.

P-Cognition Thesis

Hypothesis

Human cognitive (linguistic) capacities are constrained by polynomial time computability.

Frixione, Tractable competence. Minds and Machines, 2001.

Hintikka's branching reading

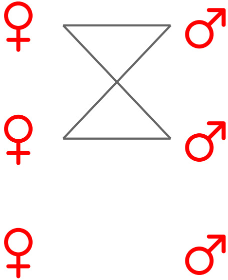
- Most girls and most boys hate each other.

$$\begin{matrix} \text{most } x : G(x) \\ \text{most } y : B(y) \end{matrix} \; H(x, y).$$

$$\exists A \exists A' [\text{most}(G, A) \wedge \text{most}(B, A') \wedge \forall x \in A \, \forall y \in A' \; H(x, y)].$$

Illustration

- Most girls and most boys hate each other.



Branching readings are intractable

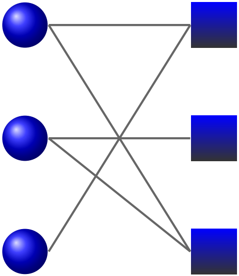
Theorem

Proportional branching sentences are NP-complete.

What about a tractable alternative?

Two-way quantification

$$(Q_1 Q_2) \wedge (Q_2 Q_1)$$



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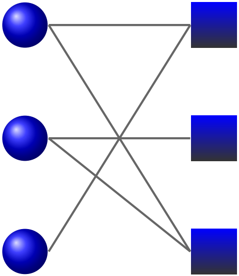

Subjects are happy to accept such interpretation.

Gierasimczuk and Szymanik, Branching quantification vs. two-way quantification, Journal of Semantics, 2009