COLLECTIVE QUANTIFICATION, TYPE-SHIFTING, AND COMPLEXITY

Jakub Szymanik

Institute for Logic, Language and Computation
Universiteit van Amsterdam

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Abstract

The common strategy in formalizing collective quantification has been to define the meanings of collective determiners using certain type-shifting operations. These type-shifting operations, i.e., lifts, define the collective interpretations of determiners systematically from the standard meanings of quantifiers. We argue that this approach is probably not expressive enough to formalize all collective quantification in natural language!

- Expressivity of a language depends on the quantifiers.
- Mainly distributive determiners are considered.
- However, plural objects are becoming important.
- E.g. in game-theory, where groups of agents are acting.

(1.) All the Knights but King Arthur *met in secret*.

(2.) Most climbers *are friends*.

(3.) John and Mary *love each other*.

(4.) The samurai *were twelve in number*.

(5.) Many girls *gathered*.

(6.) Soldiers *surrounded* the Alamo.

(7.) Tikitu and Samson *lifted* the table.

(1.) Five people lifted the table.

(1'.) $\exists^{=5} x[\mathrm{People}(x) \wedge \mathrm{Lift}(x)]$.

(1''.) $\exists X[X \subseteq \mathrm{People} \wedge \mathrm{Card}(X) = 5 \wedge \mathrm{Lift}(X)]$.

(2.) Some students played poker together.

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DEFINITION (VAN DER DOES 1992)

Fix a universe of discourse U and take any $X \subseteq U$ and $Y \subseteq \mathcal{P}(U)$. Define the existential lift Q^{EM} of a quantifier Q in the following way:

$$\mathrm{Q}^{EM}(X, Y) \text{ is true } \iff \exists Z \subseteq X[\mathrm{Q}(X, Z) \wedge Z \in Y].$$

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OBSERVATION

$(\cdot)^{EM}$ *works only for right monotone increasing quantifiers.*

(1.) No students met yesterday at the coffee shop.

— $\downarrow$MON$\downarrow \rightsquigarrow \uparrow$MON$\uparrow$

(2.) No left-wing students met yesterday at the coffee shop.

(3.) No students met yesterday at the “Che” coffee shop.

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THE TOTAL NUMBER IS MISSING

(1.) Exactly 5 students drank a whole keg of beer together.

(1'.) $(\exists^{=5})^{EM}$[Student, Drink-a-whole-keg-of-beer].

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Definition (van der Does 1992)

Let U be a universe, $X \subseteq U$, $Y \subseteq \mathcal{P}(U)$, and Q a type (1, 1) quantifier. We define the *neutral modifier*:

$$\mathrm{Q}^N[X, Y] \text{ is true} \iff \mathrm{Q}\left[X, \bigcup(Y \cap \mathcal{P}(X))\right].$$

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Monotonicity preservetion under $(\cdot)^N$

Fact (Ben-Avi and Winter 2003)

Let Q be a distributive determiner. If Q belongs to one of the classes $\uparrow$MON$\uparrow$, $\downarrow$MON$\downarrow$, MON$\uparrow$, MON$\downarrow$, then the collective determiner Q^N belongs to the same class. Moreover, if Q is conservative and $\sim$MON (MON$\sim$), then Q^N is also $\sim$MON (MON$\sim$).

DEFINITION (WINTER 2001)

For all $X, Y \subseteq \mathcal{P}(U)$ we have that

$$\mathsf{Q}^{\mathsf{dfit}}(X, Y) \text{ is true}$$

$$\Longleftrightarrow$$

$$\mathsf{Q}[\cup X, \cup(X \cap Y)] \wedge [X \cap Y = \emptyset \vee \exists W \in X \cap Y \wedge \mathsf{Q}(\cup X, W)].$$

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Monotonicity of Q	Monotonicity of Q^{dfit}	Example
$\uparrow$MON$\uparrow$	$\uparrow$MON$\uparrow$	Some
$\downarrow$MON$\downarrow$	$\downarrow$MON$\downarrow$	Less than five
$\downarrow$MON$\uparrow$	$\sim$MON$\uparrow$	All
$\uparrow$MON$\downarrow$	$\sim$MON$\downarrow$	Not all
$\sim$MON$\sim$	$\sim$MON$\sim$	Exactly five
$\sim$MON$\downarrow$	$\sim$MON$\downarrow$	Not all and less than five
$\sim$MON$\uparrow$	$\sim$MON$\uparrow$	Most
$\downarrow$MON$\sim$	$\sim$MON$\sim$	All or less than five
$\uparrow$MON$\sim$	$\sim$MON$\sim$	Some but not all

TABLE: Monotonicity under the determiner fitting operator; cf. (Ben-Avi and Winter 2003).

Definition

A distributive determiner of type (1, 1) is conservative if and only if the following holds for all M and all $A, B \subseteq M$:

$$\mathrm{Q}_M[A, B] \iff Q_M[A, A \cap B].$$

Fact

For every Q the quantifiers Q^{EM}, Q^{N}, and Q^{dfit} are not CONS.

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For every Q *the quantifiers* Q^{EM}, Q^{N}, *and* Q^{dfit} *are not CONS.*

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We say that a collective determiner Q of type $((et)(((et)t)t))$ satisfies *collective conservativity* iff the following holds for all M and all $A, B \subseteq M$:

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\begin{aligned}
\forall &= \{(M, P) \mid P = M\}. \\
\exists &= \{(M, P) \mid P \subseteq M \;\&\; P \neq \emptyset\}. \\
\mathsf{even} &= \{(M, P) \mid P \subseteq M \;\&\; \mathsf{card}(P) \text{ is even}\}. \\
\mathsf{most} &= \{(M, P, S) \mid P, S \subseteq M \;\&\; \mathsf{card}(P \cap S) > \mathsf{card}(P - S)\}. \\
\mathsf{some} &= \{(M, P, S) \mid P, S \subseteq M \;\&\; P \cap S \neq \emptyset\}.
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\exists^2 &= \{(M,P) \mid P \subseteq \mathcal{P}(M) \;\&\; P \neq \emptyset\}.\\
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Observation

SOGQs do not decide invariance properties!

Question

How invariance properties interact with definability?

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$$\text{MOST} = \{(M,P,S) \mid P,S \subseteq \mathcal{P}(M) \;\&\; \mathsf{card}(P \cap S) > \mathsf{card}(P - S)\}.$$

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Do not confuse:

- FO GQs (Lindström) with FO-definable quantifiers

  E.G. most is FO GQs but is not FO-definable.
- SO GQs with SO-definable quantifiers

  E.G. MOST is SO GQs but probably not SO-definable.

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THEOREM (KONTINEN 2002)

The extension $\mathcal{L}^*$ of first-order logic by all Lindström quantifiers cannot define the monadic second-order existential quantifier.

COROLLARY

Lindström quantifiers alone are not adequate for formalizing all natural language quantification.

EXAMPLE

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Denote by some^{EM}:

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(3'.) some^{EM} $x, X[\text{Student}(x), \text{Play}(X)]$.

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Let Q be a Lindström quantifier definable in SO. Then the collective quantifiers Q^{EM}, Q^{N}, and $\mathsf{Q}^{\mathrm{dfit}}$ are definable in SO.

PROOF.

Let us consider the case of Q^{EM}. Let $\psi(x)$ and $\phi(Y)$ be formulas. We want to express $\mathsf{Q}^{EM}x, Y(\psi(x), \phi(Y))$ in second-order logic. By the assumption, the quantifier Q can be defined by some sentence $\theta \in \mathrm{SO}[\{P_1, P_2\}]$. We can now use the following formula:

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$\square$

And this is the case for all SO-definable lifts

Theorem

Let us assume that the lift $(\cdot)^*$ and a Lindström quantifier Q are both definable in second-order logic. Then the collective quantifier Q^* is also definable in second-order logic.

(5.) Most groups of students played Hold'em together.

(5'.) MOST X, Y[Students(X), Play(Y)].

- The discussed lifts do not give the intended meaning.
- It is unlikely that *any* lift can do the job.
- Otherwise, highly unexpected things would happen!

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THEOREM

If the quantifier MOST *is definable in second-order logic, then counting hierarchy,* CH *is equal polynomial hierarchy,* PH*. Moreover,* CH *collapses to its second level.*

PROOF.

The logic FO(MOST) can define complete problems for each level of the CH (Kontinen&Niemisto'06). If MOST would be definable in SO, then FO(MOST) $\leq$ SO and therefore SO would contain complete problems for each level of the CH. This would imply that CH $=$ PH and furthermore that CH $\subseteq$ PH $\subseteq C_2P$. □

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COROLLARY

The type-shifting strategy is probably not general enough to cover all collective quantification in natural language.

CONJECTURE

The quantifier MOST *is not definable in second-order logic.*

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What is the right ontology for semantics?

- $\mathcal{L}^*$ and SO doesn't capture natural language?
- Are many-sorted (algebraic) models more plausible?
  - Type-shifting is too complex;
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Σ^1_1-THESIS

Our everyday language is semantically bounded by the properties expressible in the existential fragment of second-order logic.

- Does SOGQ "MOST" belong to everyday language?
  - Everyday language doesn't realize prop. coll. qua.
  - No need to extend the higher-order approach to prop. qua.

QUESTION

Have we just encountered an example where complexity restricts the expressibility of everyday language as suggested by Σ_1^1-thesis?

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To sum up

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- The previous attempts have relied on SO-definable GQs...
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THANK YOU FOR ATTENTION