Alternative Representations. A Case Study of Proportional Judgements

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Outline

General background

- Meaning = ‘collection of algorithms’
- Experimental testing by psycholinguists

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- Sensitivity of tasks in other domains to the manner of presentation
- Different presentations of the data create different mental representations and thereby trigger different algorithms.

Abstract

- Sensitivity of verification to a visual presentation.
- A computational model which made empirically verified predictions.
- Extending it to handle different representations.
- Predicting that they will effect working memory.
- Experiments and discussion.

Outline

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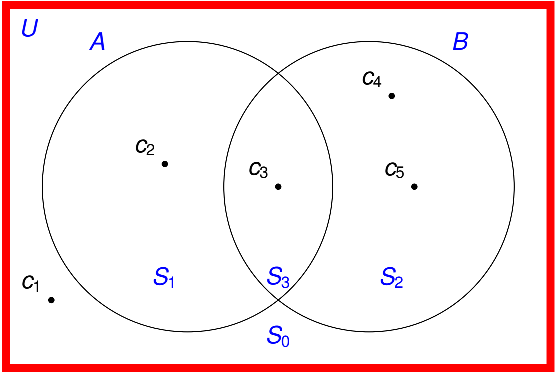
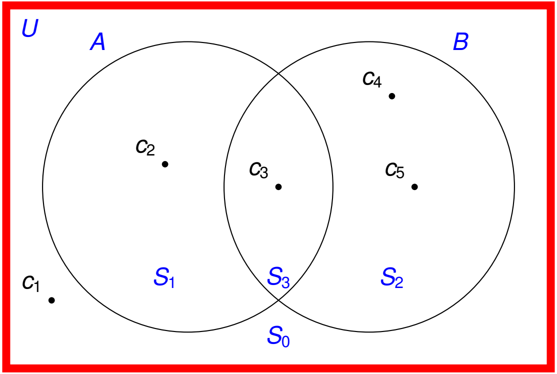
Model-theoretic view of quantifiers



How do we encode models?



This model is uniquely described by the word 01.

Step by step

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- Write a 0 for each element of $A \setminus B$ and a 1 for each element of $A \cap B$.
- Q is represented by the set of words describing all elements of the class.
- That is, Q is a formal language.

More Formally Defined

Definition

Let $\mathcal{M} = \langle M, A, B \rangle$ be a model, $\vec{a}$ an enumeration of A, and $n = |A|$. We define $\tau\left(\vec{a}, B\right) \in \{0, 1\}^n$ by

$$\left(\tau\left(\vec{a}, B\right)\right)_i = \begin{cases} 0 & a_i \in A \setminus B \\ 1 & a_i \in A \cap B \end{cases}$$

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Definition

For a type $\langle 1, 1 \rangle$ quantifier Q, define *the language of Q* as

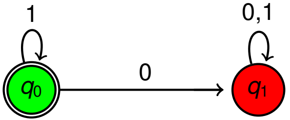
$$\mathcal{L}_Q = \left\{ s \in \{0, 1\}^* \mid \langle \#_0(s), \#_1(s) \rangle \in Q^c \right\}$$

Examples of Quantifier Languages

- $\mathcal{L}_{\text{every}} = \{w \mid \#_0(w) = 0\}$
- $\mathcal{L}_{\text{some}} = \{w \mid \#_1(w) > 0\}$
- $\mathcal{L}_{\text{most}} = \{w \mid \#_1(w) > \#_0(w)\}$

Aristotelian quantifiers

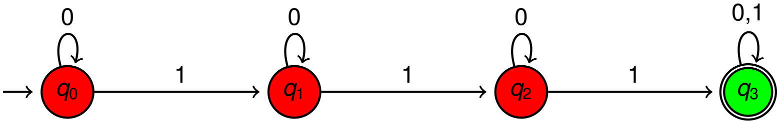
"all", "some", "no", and "not all"



Finite automaton recognizing L_{All}

Cardinal quantifiers

E.g. “more than 2”, “less than 7”, and “between 8 and 11”



Finite automaton recognizing $L_{\text{More than two}}$

Proportional quantifiers

- E.g. “most”, “less than half”.
- Most *As are B* iff card($A \cap B$) $>$ card($A - B$).
- There is no finite automaton recognizing this language.
- We need internal memory.
- A push-down automata will do.

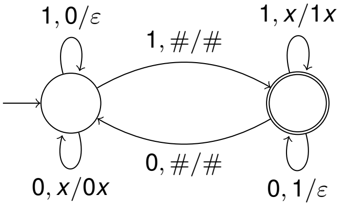


Figure : A PDA computing L_{most}.

Does it say anything about processing?

Question

Do minimal automata predict differences in verification?

Neurobehavioral studies

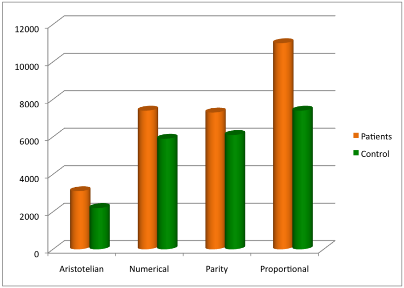
Differences in brain activity.

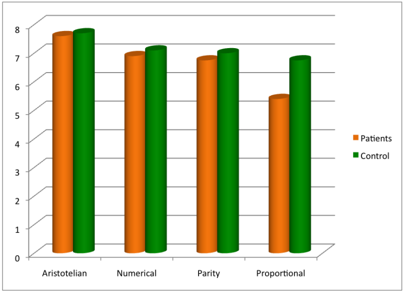
- All quantifiers are associated with numerosity:
  recruit right inferior parietal cortex.
- Only higher-order activate working-memory capacity:
  recruit right dorsolateral prefrontal cortex.

McMillan et al., Neural basis for generalized quantifiers comprehension, Neuropsychologia, 2005

Szymanik, A Note on some neuroimaging study of natural language quantifiers comprehension, Neuropsychologia, 2007

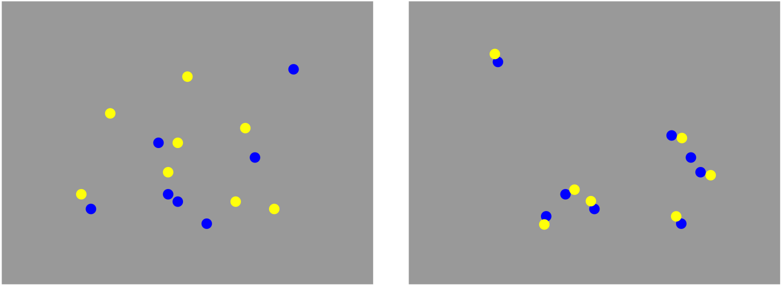
Schizophrenic patients





Zajenkowski et al., A computational approach to quantifiers as an explanation for some language impairments in schizophrenia, Journal of Communication Disorders, 2011.

What about input variations?



Model saliency

- We take models of the form $\langle M, A, B, R\rangle$ where $R \subseteq M \times M$.
- τ' maps such models into an alphabet containing *pairs* of symbols in addition to 0s and 1s.
- τ' will map all pairs in R to pairs of symbols in the natural way,
- e.g., $\langle a, b\rangle \in R$ where $a \in A \cap B$ and $b \in A \setminus B$, then $\langle a, b\rangle$ will get mapped to $\langle 1, 0\rangle$.
- Any elements of the model that are not paired will get mapped to 0 or 1.
- L'_{most} is the set of strings where the only pairs are $\langle 1, 0\rangle$ or $\langle 0, 1\rangle$ and all individual symbols are 1s.
- This language, however, is paradigmatically regular.

So theoretically it should make a difference . . .

Outline

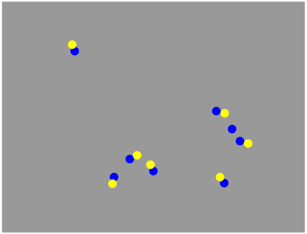
Method

4 experiments:

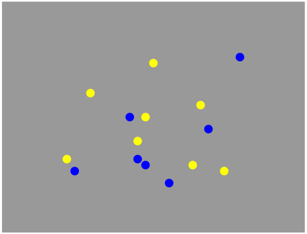
1. Are more than half of the dots yellow?
2. Are most of the dots yellow?
3. Are most of the letters ‘O’?
4. Are more than half of the letters ‘E’?

Manipulations:

- random and paired
- 8/7, 9/8 and 10/9 proportions



Exp. 1, 2



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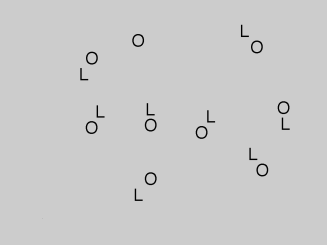
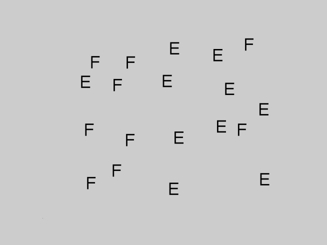


Exp. 3



Exp. 4

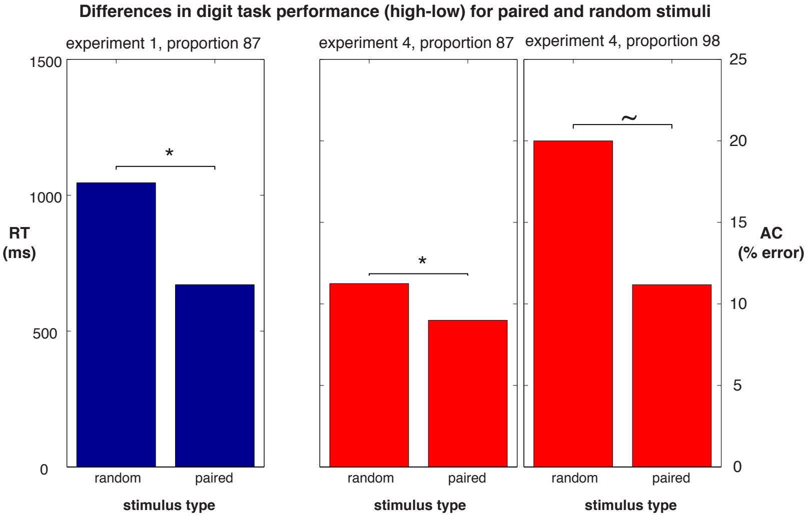
Manipulating WM

- A digit recall task.
- A string of 5 digits for 1500ms.
- Probing one digit.
- Blocks with low memory condition: the same sequence of digits.
- Blocks with high memory condition: the digits were randomized.

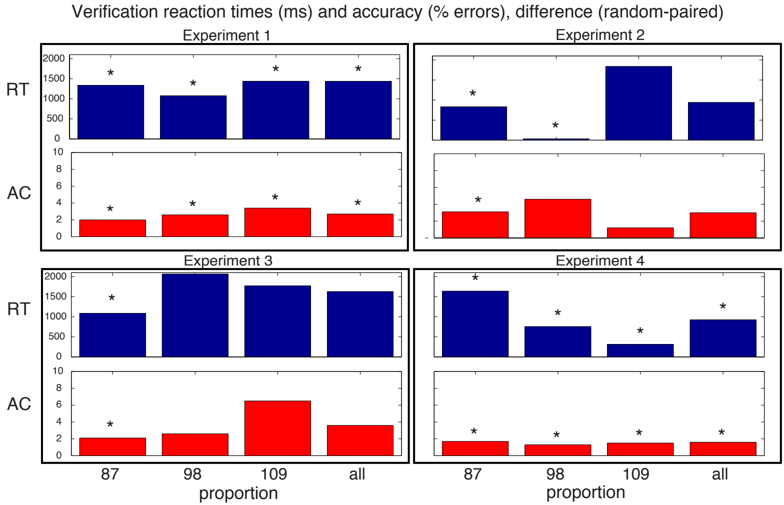
Participants

- M'Turk with HIT approval rate of at least 99%.
- Exp. 1: $N = 59$, 28 male, age 20–59 ($M = 33$, $SD = 9.9$)
- Exp. 2: $N = 57$, 28 male, age 20–68 ($M = 35$, $SD = 9.6$)
- Exp. 3: $N = 56$, 18 male, age 19–75 ($M = 40$, $SD = 14$)
- Exp. 4: $N = 54$, 27 male, age 20–69 ($M = 35$, $SD = 12$)

Effects of the interaction of stimulus type and WM in the digit recall task



Effects of stimulus type on verification RT and accuracy



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  - making a speed-accuracy tradeoff
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- Using WM to distinguish verification strategies

Outlook

- Back-and-forth between logic and cognition
- Logic brings complexity classification
- Cognitive science representation/strategies
- We need put it together in a form of a cognitive model