Using the intrinsic complexity of turn-based games to predict participants' decision times

Jakub Szymanik

Institute for Logic, Language and Computation
University of Amsterdam

joint work with Ben Meijering and Rineke Verbrugge

Higher-order social cognition

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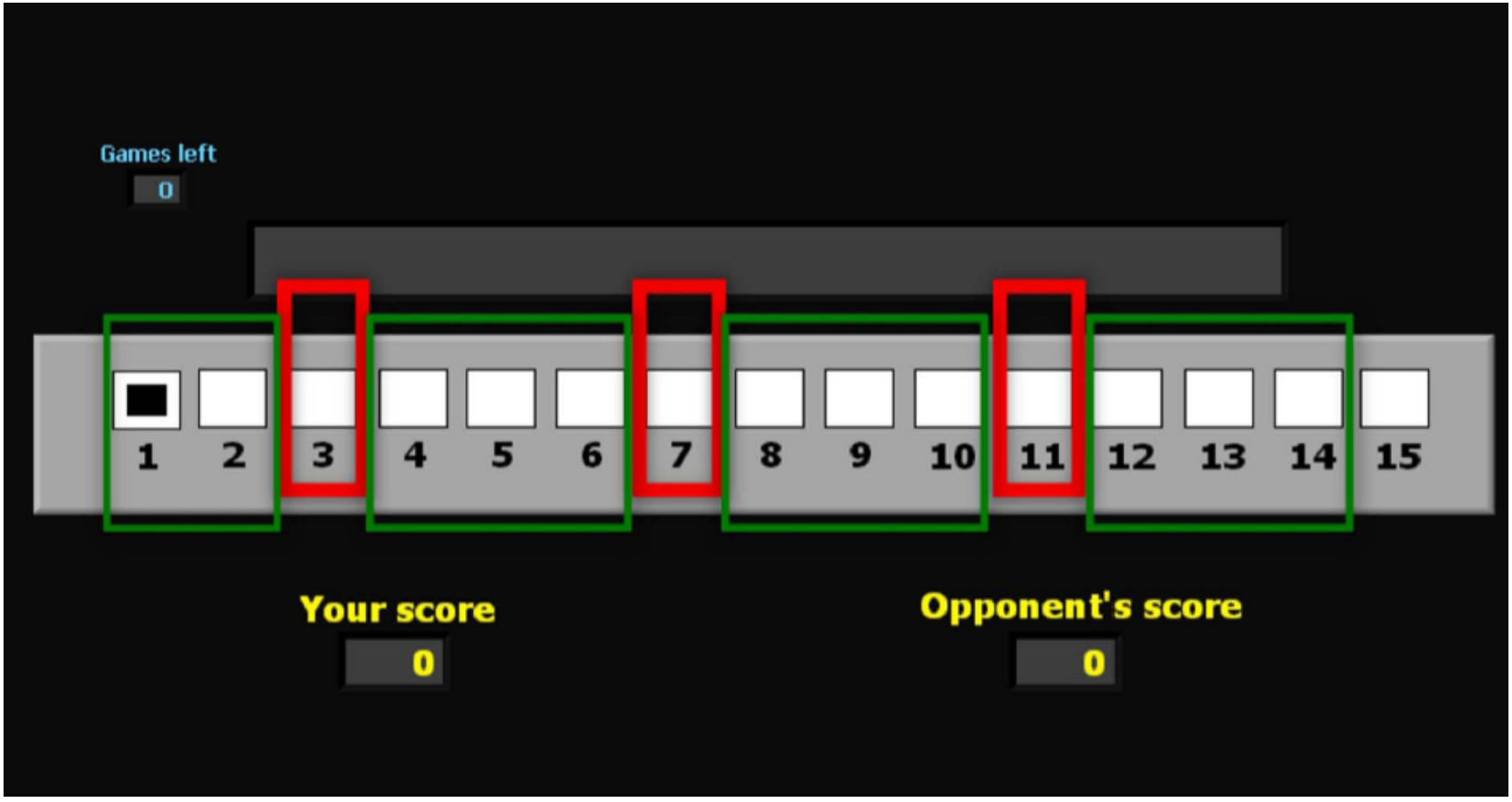
3. Backward induction: tells us which sequence of actions will be chosen by agents that want to maximize their own payoffs, assuming common knowledge of rationality.
4. Turn-based games have been extensively studied in psychology

HIT-N Game



Gneezy et al. Experience and insight in the race game, 2010

Hawes et al. Experience and abstract reasoning in learning backward induction, 2012

Turn-based games

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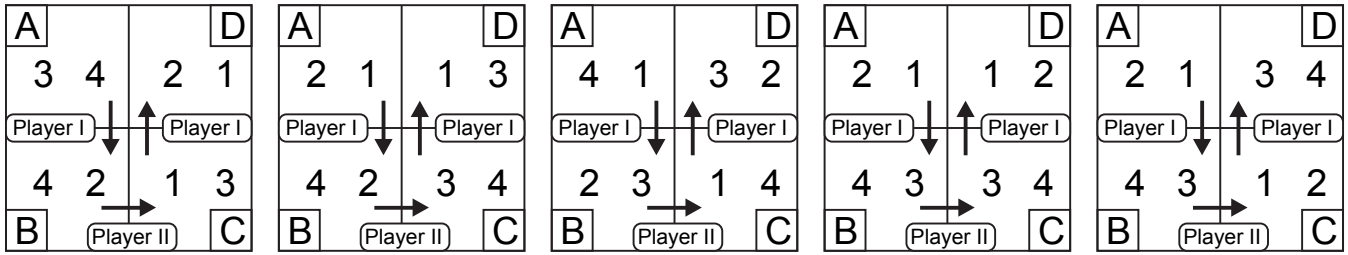
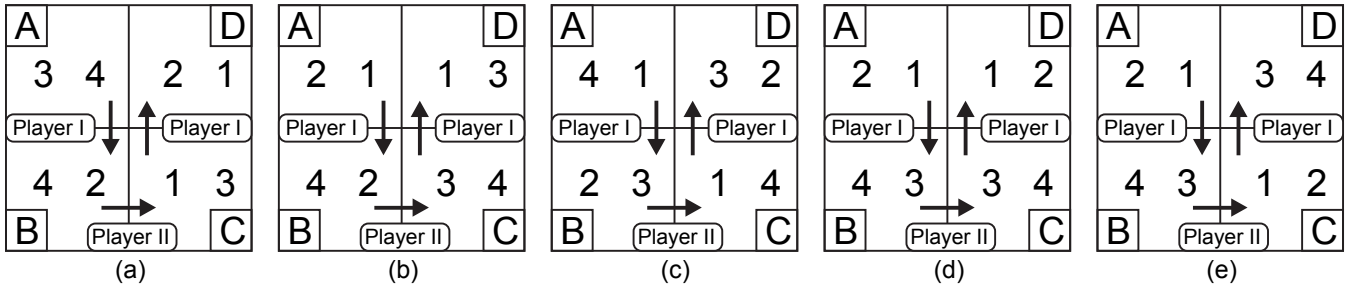
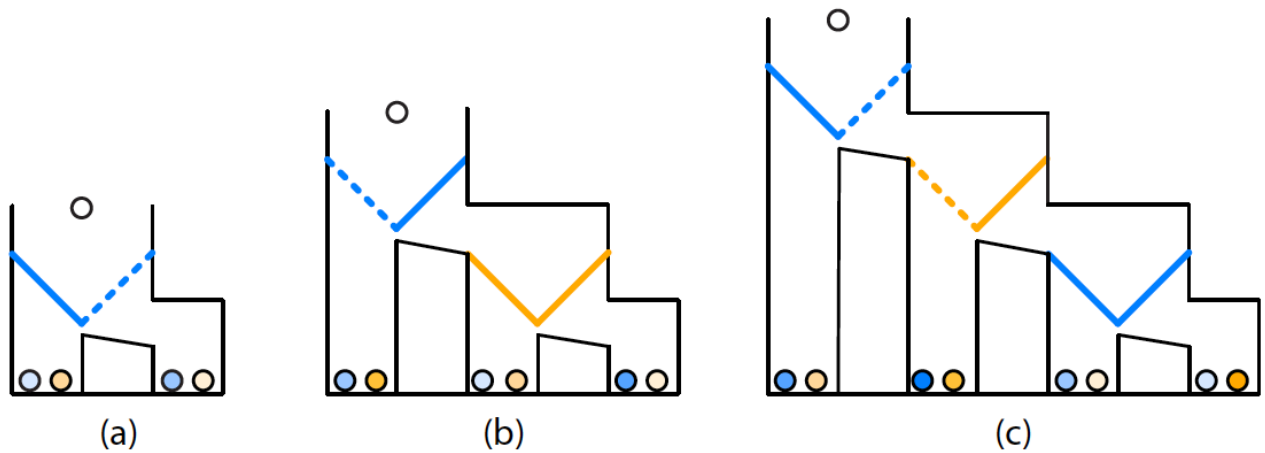


Turn-based games

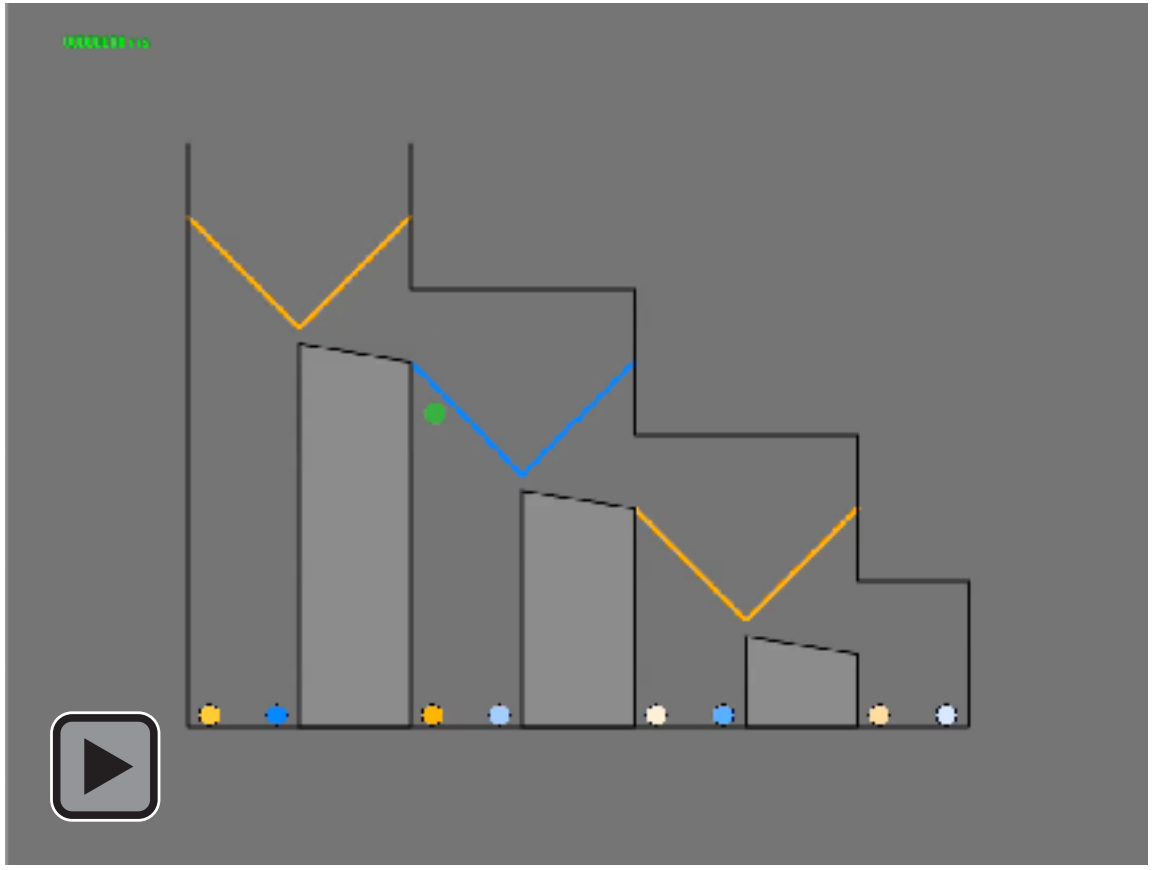





Hedden & Zhang What do you think I think you think?, 2002


Meijering et al., The facilitative effect of context on second-order social reasoning, 2010

Subjects don’t use BI

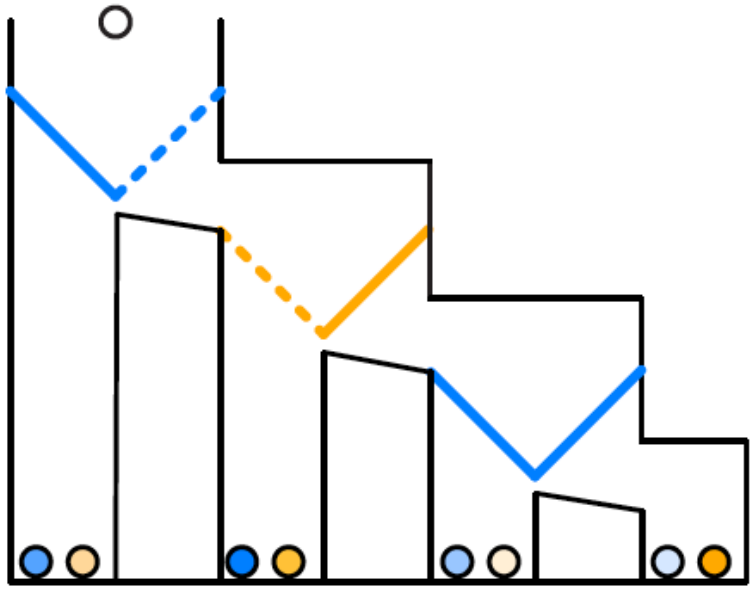


Project

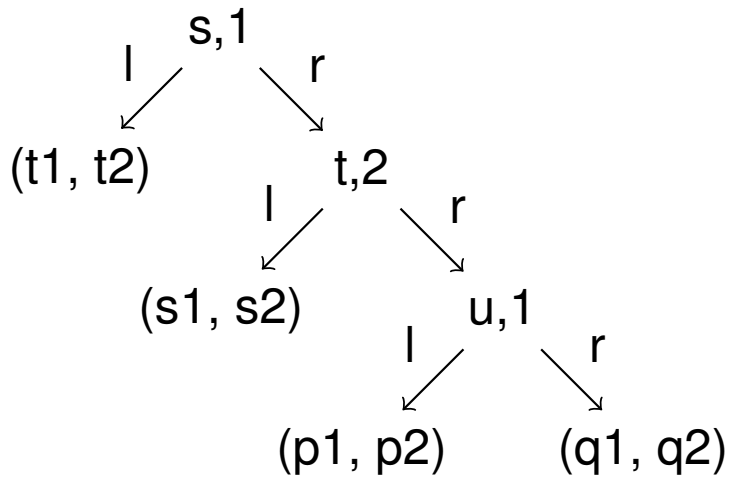
1. What is the complexity of the computational problem?
2. What makes certain trials harder than others?
3. What is the connection with logic?
4. What is the connection with game-theory?

$\hookrightarrow$ human reasoning strategies and bounded rationality

Marble Drop Game



Logical analysis: MDG decision trees



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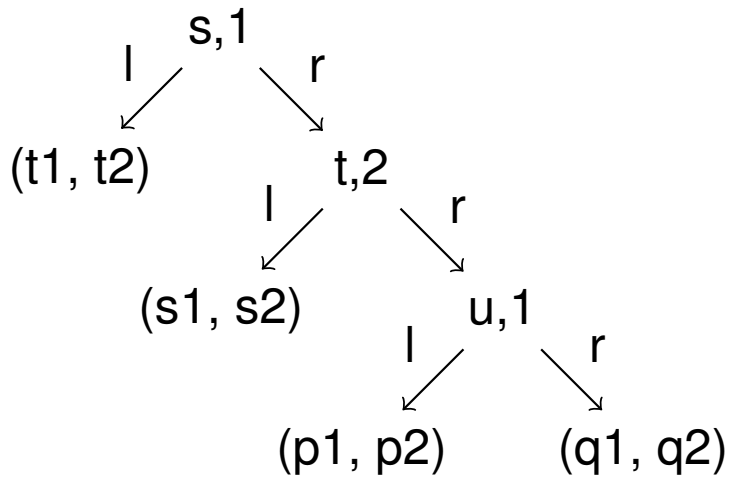
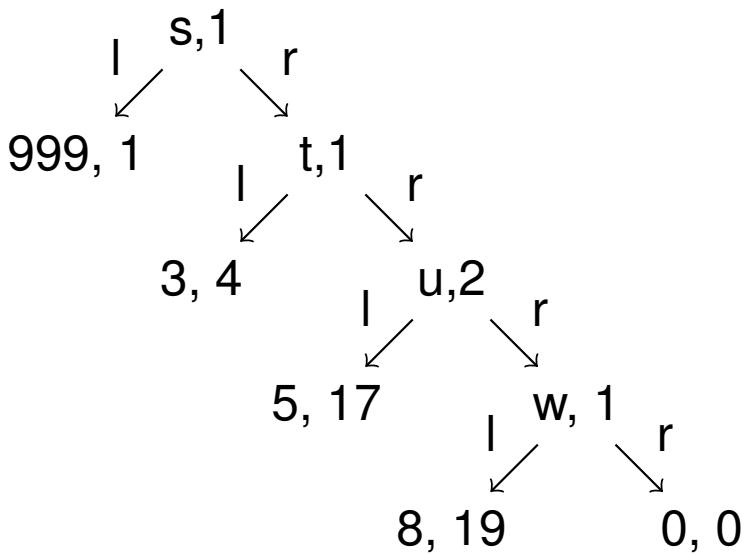
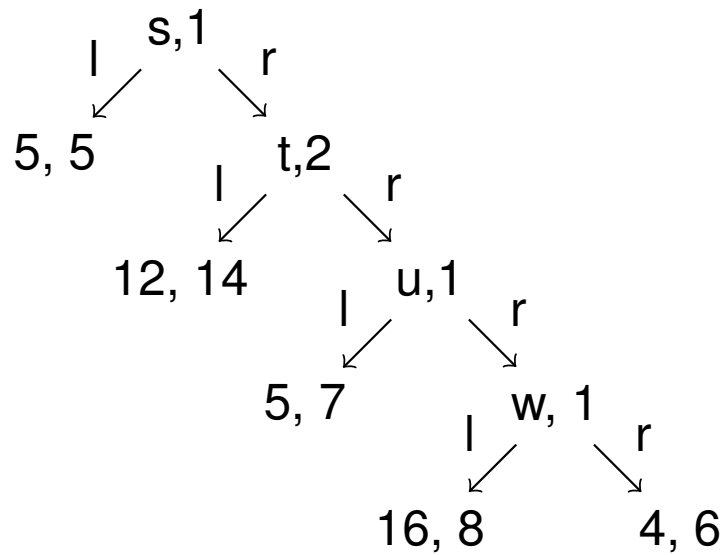


Definition

G is generic, if for each player, distinct end nodes have different pay-offs.

Pay-off structure





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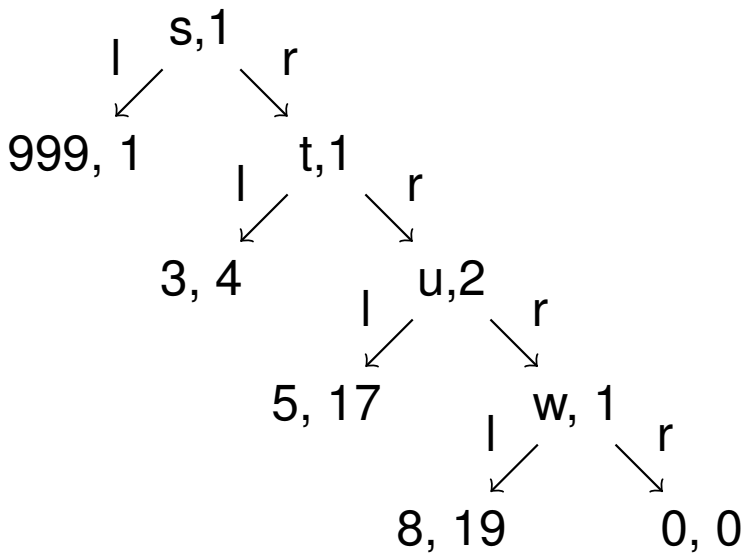
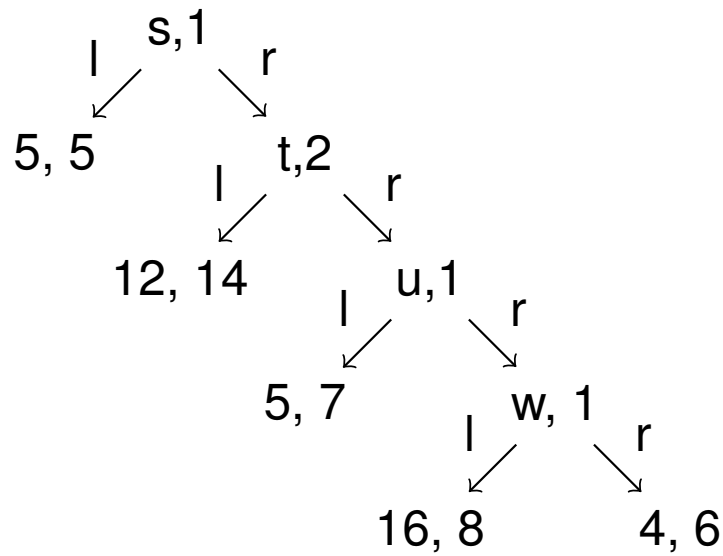
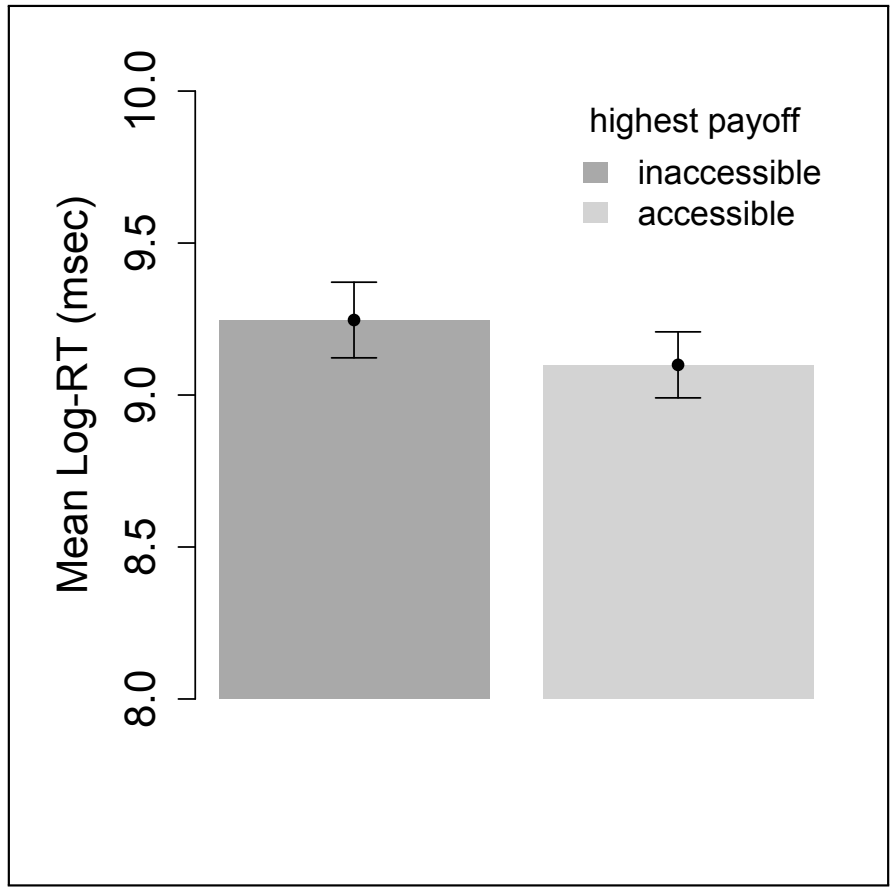




Forward reasoning + backtracking is consistent with eye-tracking study.

Forward Reasoning + Backtracking, FRB



FRB is indeed rational

For an average random game with 3 decision points, the forward reasoning plus backtracking algorithm needs fewer computation steps to yield a correct solution than backward induction.

Table : Cross-table of payoff structures and the necessary number of steps when using forward reasoning with backtracking on all 576 possible experimental pay-off structures.

# of steps	1	2	4	5	6	8
# of payoff structures	288	72	48	56	16	96

On average: BI=6 and FRB=3

Descriptive complexity: alternation type

Definition

Let's assume that the players strictly alternate in the game. Then:

1. In a Λ_1^i tree all the nodes are controlled by Player i.
2. In a Λ_k^i tree, k-alternations, starts with an ith Player node.

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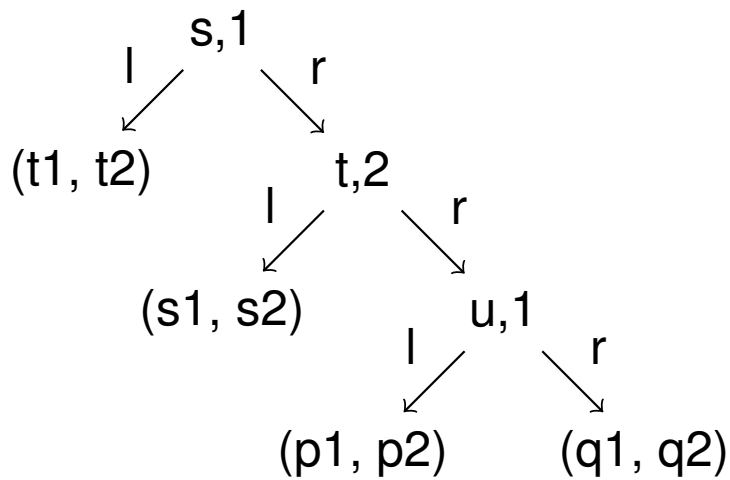

Figure : Λ_3^1 -tree

Recall, . . .

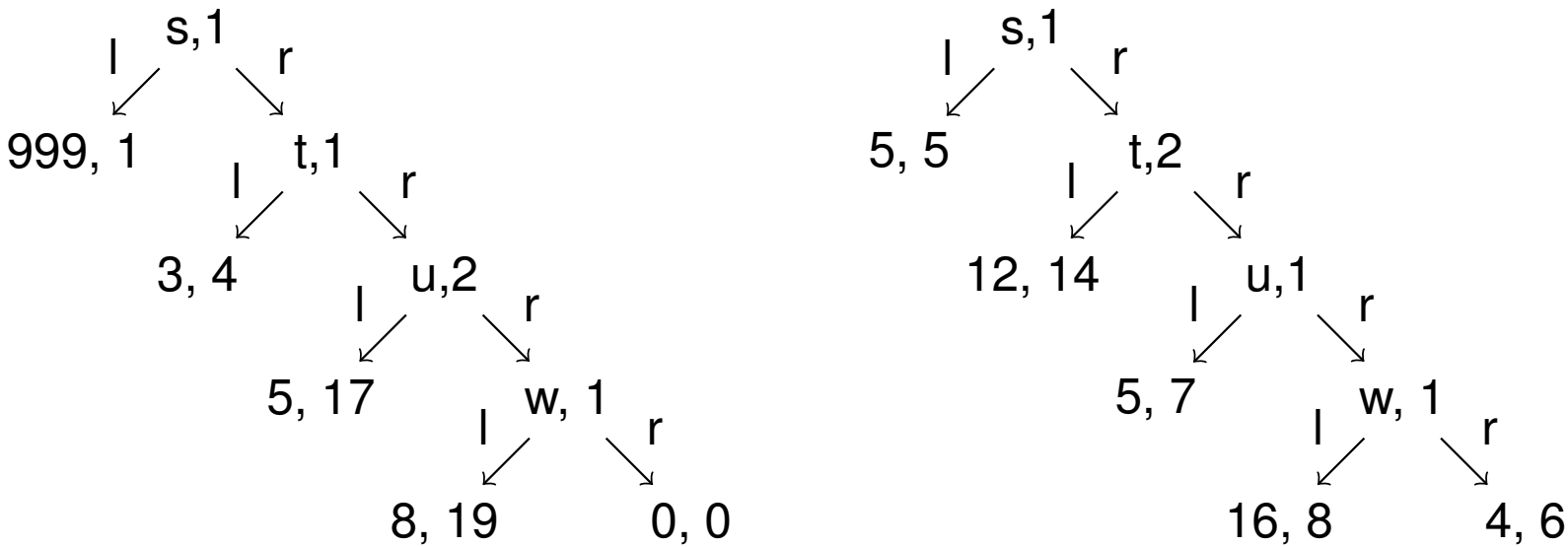

Figure : Two Λ_3^1 trees.

T^--example

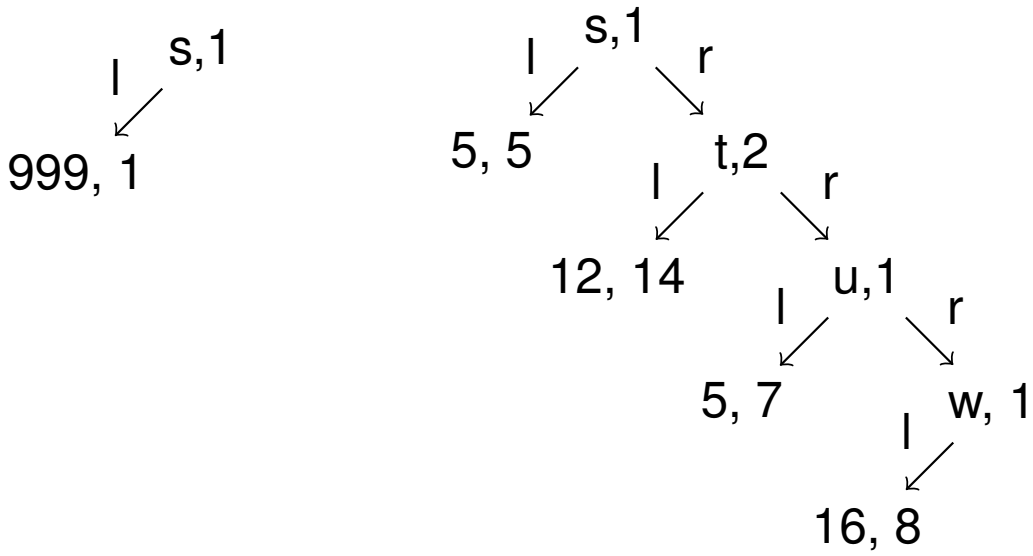

Figure : Λ_1^1 tree and Λ_3^1 tree

T^-

Definition

If T is a generic game tree with the root node controlled by Player 1 (2) and n is the highest pay-off for Player 1 (2), then T^- is the minimal subtree of T containing the root node and the node with pay-off n for Player 1 (2).

Conjecture

Observation

If T_1 is accessible and T_2 is inaccessible then $T_1^- < T_2^-$.

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Conjecture

Let us take two MDG trials T_1 and T_2. T_1 is easier for participants than T_2 if and only if T_1^- is lower in the tree alternation hierarchy than T_2^-.

FRB and structural complexity

Hypothesis

Let us take two MDG trials T_1 and T_2. Forward reasoning plus backtracking yields a correct solution for T_1 faster than T_2 if and only if T_1^- is lower in the tree alternation hierarchy than T_2^-.

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Table : Output of full-factorial linear mixed-effects model with factors Accessibility (A), Steps of forward reasoning with backtracking (FRB) applied to the subset of actually presented experimental games.

Parameter	Estimate	St. Error	t-value	p-value
a) Accessible	-0.689147	0.271256	-2.54	.000
b) FRB	0.008767	0.034930	0.25	.418
c) A:FRB	0.084336	0.037277	2.26	.000

- FRB steps are a good predictor of RT.
- RT decreases for ‘accessible games’.
- No significant effect for ‘inaccessible games’.
- RT increases with each additional FRB step in ‘accessible games’.

Summary of the results

- Structural properties responsible for the cognitive difficulty
- Results generalized to other turn-based games

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- Structural properties responsible for the cognitive difficulty
- Results generalized to other turn-based games
- FRB avoids higher-order reasoning
- FRB is computationally optimal

Thank you

E-mail: jakub.szymanik@gmail.com

URL: http://www.jakubszymanik.com/