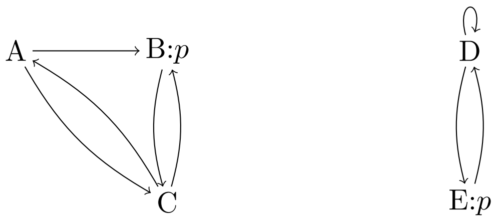
Homework #2

November 20, 2009

The assignments are due Friday, November 27 at 11AM in class. All questions have equal weight. Good luck!

Exercise 1 Recall exercise 3 from the previous homework.

1. Give a winning strategy for Duplicator in the bisimulation game played on the following models from the match between points A and D.



2. Give a winning strategy for Spoiler in the bisimulation game played on the following models from the match between points A and D. How many moves does Spoiler need?

Exercise 2 Consider the following new operator $\Box^{\leftarrow}$ defined on relational structures $M = (W, R, V)$ in the following way:

$$M, w \models \Box^{\leftarrow}\varphi \text{ iff for all } v \in W, \text{ if } vRw \text{ then } M, v \models \varphi$$

Prove that $\Box^{\leftarrow}$ is not definable in the basic modal language.

Exercise 3

1. Prove the following formula in the system K: $\Box(\varphi \wedge \psi) \iff (\Box\varphi \wedge \Box\psi)$.
2. Prove the formula $\Box\Diamond\Box\Diamond\varphi \iff \Box\Diamond\varphi$ in the logic S4.

Exercise 4 Prove soundness theorem for K:

$$\text{If } \vdash_K \varphi \text{ then } \varphi \text{ holds on all frames.}$$

Exercise 5 We say that frame $F = (W, R)$ is Euclidean if the relation R is Euclidean (for all $x, y, z \in W$ if xRy and xRz then yRz). Prove that F is Euclidean iff $F \models \Diamond\varphi \implies \Box\Diamond\varphi$.