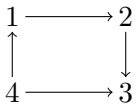
Homework #1

November 13, 2009

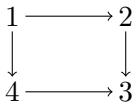
The assignments are due Friday, November 20 at 11AM in class. All questions have equal weight. Have fun!

Exercise 1

1. For each point in the following model give a modal formula that is only true at it. Hint: use $\top$ and $\bot$.

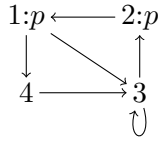


2. Which states can you uniquely characterize by modal formulas in the next model? Give the formulas and explain your answer.



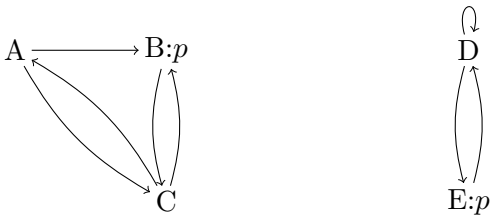
Exercise 2

1. Determine in which states of the following model the modal formula $\Diamond\Box\Diamond\, p$ is true.
2. Give a complete game tree for the evaluation game for $\Diamond\Box\Diamond\, p$ in this model starting in state 1.



Exercise 3

1. Draw a bisimulation between the following two models, connecting points A and D. Check that it satisfies the definition.



2. Show that there is no bisimulation between the points A and D in the next two models by giving a modal formula true in only one of them.

Exercise 4

1. Prove that:
   (a) formula $\Diamond(P \implies Q) \implies (\Box P \implies \Diamond Q)$ is valid;
   (b) formula $\Box(\Box P \implies Q) \vee \Box(\Box Q \implies P)$ is not valid.
2. Compute the modal depth for both formulas.

Exercise 5 Prove the following. Let f be an isomorphism between models $M = (W, R, V)$ and $M' = (W', R', V')$. Then for all basic modal formulas φ, and all points w in M, we have that:

$$M, w \models \varphi \text{ iff } M', f(w) \models \varphi$$

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